

*An Introduction to Real Analysis* gives students of mathematics and related sciences an introduction to the foundations of calculus, and more generally, to the analytic way of thinking. The authors' style is a mix of formal and informal, with the intent of illustrating the practice of analysis and emphasizing the process as much as the outcome.

The book is intended for use in a one- or two-term course for advanced undergraduates in mathematics and related fields who have completed two or three terms of a standard university calculus sequence.

ISBN 978-1-4704-7421-8



9 781470 474218

AMSTEXT/65



For additional information  
and updates on this book, visit  
[www.ams.org/bookpages/amstext-65](http://www.ams.org/bookpages/amstext-65)



This series was founded by the highly respected  
mathematician and educator, Paul J. Sally, Jr.



Preface	xi
Chapter 1. The Real Numbers	1
1.1. The natural numbers	1
1.2. The integers	6
1.3. The rational numbers	7
1.4. Least upper bounds and greatest lower bounds	14
1.5. The real numbers	17
*1.6. Rational powers of real numbers	22
*1.7. Real powers of real numbers	29
*1.8. Euclidean space	34
Chapter 2. Sequences and Series	37
2.1. Metric spaces	37
2.2. Sequences	39
2.3. Sequences of real numbers	40
2.4. The density of the rational numbers in the real numbers	47
2.5. Subsequences	49
2.6. Series	56
2.7. Convergence criteria	60
2.8. The decimal expansion of a real number	69
*2.9. Binary and ternary representations	71
*2.10. Rearrangements	72
Chapter 3. The Topology of the Real Line	77
3.1. Closed sets and open sets in $\mathbb{R}$	77



3.2.	Compactness	85
3.3.	Small sets in $\mathbb{R}$	91
Chapter 4.	Continuous Functions	101
4.1.	Limits of functions	101
4.2.	Continuity	107
4.3.	Points of discontinuity	111
4.4.	Continuous functions on bounded intervals	116
*4.5.	The space $C[0, 1]$	118
Chapter 5.	Sequences and Series of Functions	121
5.1.	Sequences of functions	121
*5.2.	Completeness and separability of $C[0, 1]$	128
5.3.	Series of functions	133
5.4.	Power series	135
Chapter 6.	Differentiation	141
6.1.	The derivative	141
6.2.	The arithmetic of differentiable functions	146
6.3.	Functions that are differentiable in an interval	149
6.4.	Derivatives of power series	155
6.5.	Taylor's theorem	157
6.6.	Trigonometric functions (without circles)	161
Chapter 7.	The Riemann Integral	167
7.1.	The Riemann integral	168
7.2.	Properties of Riemann integrals	174
7.3.	Integrable functions	180
7.4.	Improper integrals	187
7.5.	The Riemann-Stieltjes Integral	188
Chapter 8.	The Lebesgue Integral	195
8.1.	Lebesgue measure on the real line	196
8.2.	Measurable functions	212
8.3.	The Lebesgue integral	218
8.4.	Convergence theorems	225
8.5.	The fundamental theorem of calculus for the Lebesgue integral	229
8.6.	The space $\mathcal{L}^1(\mathbb{R})$	238
Appendix A.	Set theory and constructing $\mathbb{R}$	245
1.	Some set theory, in a (very small) nutshell	245



2.	Cardinality (in its own small nutshell)	251
3.	A construction of the real numbers	256
Index		263