

The book introduces complex analysis as a natural extension of the calculus of real-valued functions. The mechanism for doing so is the *extension theorem*, which states that any real analytic function extends to an analytic function defined in a region of the complex plane. The connection to real functions and calculus is then natural. The introduction to analytic functions feels intuitive and their fundamental properties are covered quickly. As a result, the book allows a surprisingly large coverage of the classical analysis topics of analytic and meromorphic functions, harmonic functions, contour integrals and series representations, conformal maps, and the Dirichlet problem. It also introduces several more advanced notions, including the Riemann hypothesis and operator theory, in a manner accessible to undergraduates. The last chapter describes bounded linear operators on Hilbert and Banach spaces, including the spectral theory of compact operators, in a way that also provides an excellent review of important topics in linear algebra and provides a pathway to undergraduate research topics in analysis.



The book allows flexible use in a single semester, full-year, or capstone course in complex analysis. Prerequisites can range from only multivariate calculus to a transition course or to linear algebra or real analysis. There are over one thousand exercises of a variety of types and levels. Every chapter contains an essay describing a part of the history of the subject and at least one connected collection of exercises that together comprise a project-level exploration.

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