

Contents

<i>Preface</i>		<i>page</i> xix
<i>Acknowledgments</i>		xxi
1 The Failure of Classical Physics		1
1.1 Problems		1
Problem 1	Black-Body Radiation Spectrum	1
Problem 2	Compton Scattering for an Electron in Motion in the Lab Frame	5
Problem 3	The Thomson Model of the Atom and Rutherford's Experiment	9
Problem 4	The Stability Problem for the Rutherford Model of the Atom	11
Problem 5	Bohr's Calculation of the Energy Spectrum of the Hydrogen Atom	12
Problem 6	The Bohr–Sommerfeld Quantization Rule and the Harmonic Oscillator Energy Spectrum	12
Problem 7	An Application of the Bohr–Sommerfeld Quantization Rule to the Hydrogen Atom	14
Problem 8	Heat Capacity of Solids	15
2 Wave–Particle Duality and Wave Mechanics		18
2.1 Wave Packets		18
2.2 Wave Mechanics		20
2.3 Problems		23
Problem 1	Group and Phase Velocities of the Matter Wave for a Relativistic Particle	23
Problem 2	Diffraction of Neutrons by a Linear Chain of Nuclei	24
Problem 3	Bragg Reflection with Index of Refraction	24
Problem 4	Shape of a Wave Packet in Three Dimensions as a Function of Time	26
Problem 5	The Green's Function for a Free Particle	27
Problem 6	Dominant Contribution to the Free-Particle Wave Packet	28
Problem 7	Time Evolution of a Gaussian Free-Particle Wave Packet	30
Problem 8	Probability Density and Probability Current Density as Expectation Values of Operators	33
Problem 9	Wigner Distribution	34
3 Schrödinger Equation; Uncertainty Relations		36
3.1 Schrödinger Equation for a Particle in a Potential		36
3.2 Heisenberg's Uncertainty Relations		38
3.3 Problems		39

Problem 1	Average Values of Position and Momentum for a Wave Function $e^{-i\langle p \rangle x/\hbar} \psi(x + \langle x \rangle)$	39
Problem 2	Probability and Current Densities for a Particle in a Potential	39
Problem 3	Probability and Current Densities for a Charged Particle in an Electromagnetic Field	40
Problem 4	Gram–Schmidt Orthogonalization	41
Problem 5	Heisenberg’s Uncertainty Relations: A Derivation	43
Problem 6	Heisenberg’s Uncertainty Relations: An Alternative Derivation	44
Problem 7	Schrödinger Equation in the p -Representation	45
Problem 8	Averages and Variances of Position and Momentum Operators for Gaussian Wave Packet	47
Problem 9	Formulation of the Schrödinger Equation for $\Psi(\mathbf{r}, t) = e^{iS(\mathbf{r}, t)/\hbar}$	49
Problem 10	Time Evolution of the Averages of the Position and Momentum Operators	52
Problem 11	Time Evolution of the Average of a Generic Time-Dependent Operator	55
Problem 12	Charged Particle in an Electromagnetic Field: Lorentz Force	55

4 The One-Dimensional Schrödinger Equation; Bound States 61

4.1	Nature of the Energy Spectrum and General Properties of the Eigenfunctions	61
4.2	Parity	63
4.3	Problems	64
Problem 1	No Acceptable Solutions for Energies Less Than the Minimum of $v(x)$	64
Problem 2	Degeneracy of the Energy Eigenvalues Corresponding to a Potential $v(x)$	65
Problem 3	Nodes of Excited-State Wave Functions	65
Problem 4	Coordinate- and Momentum-Space Bound-State Wave Functions in a δ -Function Potential	66
Problem 5	The Schrödinger Equation in Momentum Space for an Attractive δ -Function Potential	68
Problem 6	Inequality for Ground-State Energies Corresponding to Potentials $v(x) \leq \bar{v}(x)$	70
Problem 7	Particle in an Asymmetric One-Dimensional Potential Well	71
Problem 8	Particle in a Potential $v(x) = \infty$ for $x < 0$ and $v(x) = -v_0 \delta(x - a)$ for $x > 0$	74
Problem 9	Particle in a Potential $v(x) = -v_0 [\delta(x) + \delta(x - a)]$: The Ionized H_2^+ Molecule	75
Problem 10	Particle in a Potential $v(x) = -v_0 \delta(x)$ for $ x < a/2$ and $v(x) = \infty$ for $ x > a/2$	78
Problem 11	Particle in a Potential $v(x) = v_0 \theta(a - x) - w_0 \delta(x)$	81
Problem 12	The WKB Approximation for Bound-State Solutions of the Schrödinger Equation	83
Problem 13	Bound States for an Infinite Barrier at $x = 0$ and a Finite Barrier for $a < x < b$	90

5	Scattering in One Dimension	94
5.1	Description in Terms of Wave Packets	95
5.2	An Alternative Treatment	97
5.3	Problems	98
Problem 1	Scattering in a Repulsive δ -Function Potential	98
Problem 2	Study of the Potential Step	102
Problem 3	Linear Potential in Momentum Space	108
Problem 4	Finite Barrier: Tunneling, Transmission, and Transit Time	110
Problem 5	Tunneling in the Limit of a High and/or Wide Barrier	114
Problem 6	Bound- and Scattering-State Problems in a Potential $v(x) = v_0 \theta(-x) - w_0 \delta(x)$	116
Problem 7	Wave Functions for a Potential $v(x < 0) = \infty$ and $v(x > 0) = -v_0 \theta(a - x)$	118
Problem 8	Transmission and Reflection Coefficients in WKB Approximation with Application	122
Problem 9	Scattering and Resonances in a Potential $v(x) = \infty$ for $x < 0$ and $v_0 \delta(x - a)$ for $x > 0$	125
Problem 10	Scattering in a Potential with an Infinite Barrier at $x=0$ and a Finite Barrier for $a < x < b$	129
Problem 11	Reflection and Transmission for a Particle Confined to the xy -Plane with $v(x, y) = v_0 \theta(x)$	134
Problem 12	Reflection and Transmission at Two Repulsive δ -Function Potentials	137
Problem 13	Transmission through Equally Spaced Repulsive δ -Function Potentials	139
Problem 14	A General Treatment of Scattering: S -Matrix	144
Problem 15	Scattering in a Parity-Invariant Potential: Phase-Shift Method	148
Problem 16	Application to Scattering in a Repulsive δ -Function Potential	149
Problem 17	Reflection and Transmission in a Generic Potential	152
6	Mathematical Formulation of Quantum Mechanics	155
6.1	Hilbert Space of Square-Integrable Functions	155
6.2	Abstract Hilbert Space	157
6.3	Representations	160
6.4	Hermitian Operators and Observables	162
6.5	The Coordinate and Momentum Representations	164
6.6	Tensor Products	165
6.7	Problems	167
Problem 1	The Set of Square-Integrable Functions Forms a Linear Vector Space	167
Problem 2	The Parity Operator as a Hermitian and Unitary Operator	167
Problem 3	Properties of the Projection Operator	168
Problem 4	A Projection Operator onto a State $ \psi\rangle$ in a Three-Dimensional State Space	168
Problem 5	Properties of the Operator $\hat{O}_{mn} = \phi_m\rangle\langle\phi_n $	169
Problem 6	A Unitary Operator	170
Problem 7	Exponentiating the Pauli Matrix σ_y	171

Problem 8	The Transformation Matrix Relating Two Bases is Unitary	171
Problem 9	The Momentum Operator in the Coordinate Representation	172
Problem 10	Momentum and Hamiltonian Eigenvalue Problems in the Coordinate Representation	173
Problem 11	The Position Operator in the Momentum Representation	174
Problem 12	The Hamiltonian Eigenvalue Problem in the Momentum Representation	174
Problem 13	Some Consequences of the Commutation Relation between \hat{x} and \hat{p}	175
Problem 14	Trace of an Operator	176
Problem 15	Properties of Eigenvalues and Eigenstates of a Hermitian Operator	177
Problem 16	Decompositions of Hermitian Operator in Terms of Its Eigenvalues or Eigenstates	177
Problem 17	Basis of Simultaneous Eigenstates for Two Observables, Commutativity	178
Problem 18	Simultaneous Eigenstate of Two Anticommuting Observables	179
Problem 19	Normal Operators and Associated Eigenvalues	180
Problem 20	A Model Hamiltonian	182
Problem 21	A Simple Two-State Hamiltonian	183
Problem 22	Eigenvalues and Eigenvectors of a Two-State Hamiltonian	184
Problem 23	Two Observables in a Three-Dimensional State Space: An Example	185
Problem 24	Model for a Planar Molecule	187
Problem 25	Derivation of Formulae Relating to Exponentials of Operators	190

7 Physical Interpretation: Postulates of Quantum Mechanics 192

7.1	Time Evolution Operator and Time Dependence of Expectation Values	194
7.2	Schrödinger and Heisenberg Pictures	196
7.3	Problems	196
Problem 1	The Probability $p(a_i)$ is Independent of the Basis Adopted in a Degenerate Subspace	196
Problem 2	Explicit Time Dependence of $\langle \hat{A}(t) \rangle$	197
Problem 3	Time–Energy Relation	198
Problem 4	Measurements on a Generic Wave Function $\psi(\mathbf{r})$	198
Problem 5	Measurements of Non-Commuting Observables	200
Problem 6	Energy Measurements for a Particle in a One-Dimensional Infinitely Deep Well	202
Problem 7	Energy Measurements for a Particle in a Two-Dimensional Infinitely Deep Well	203
Problem 8	Measurements of Three Observables in a Three-Dimensional State Space: Example	207
Problem 9	Consequences of a Sudden Change in the Potential	211
Problem 10	Time Evolution of $\langle x(t) \rangle$ and $\langle p(t) \rangle$ in a One-Dimensional Linear Potential	216
Problem 11	The Virial Theorem in One Dimension	217
Problem 12	Interaction Representation	218
Problem 13	Two-Flavor Neutrino Oscillations	220

8	The Harmonic Oscillator	224
8.1	Lowering and Raising Operators and the Number Operator Representation	224
8.2	Wave Functions of the Harmonic Oscillator	226
8.3	Problems	227
Problem 1	Some Properties of the Raising and Lowering Operators	227
Problem 2	Eigenvalue Problem for \hat{x} in the N -Representation	228
Problem 3	Position Measurement and Subsequent Time Evolution of a Harmonic Oscillator	231
Problem 4	N Uncoupled Harmonic Oscillators	233
Problem 5	Energy Measurements on Two Uncoupled Harmonic Oscillators	236
Problem 6	Time Evolution of Position and Momentum Operators in Heisenberg Picture	240
Problem 7	Two Uncoupled Harmonic Oscillators and the Exchange Operator	243
Problem 8	Momentum-Space Eigenfunctions of the Harmonic Oscillator	245
Problem 9	A Hamiltonian Consisting of a Harmonic Oscillator Plus a δ -Function Potential	248
Problem 10	Coherent States	252
Problem 11	Model of a One-Dimensional Crystal	260
9	Particle in a Central Potential; Orbital Angular Momentum	270
9.1	Orbital Angular Momentum	270
9.2	The Spectrum of \mathbf{L}^2 and L_z	272
9.3	Spherical Harmonics	274
9.4	Problems	276
Problem 1	Commutation Relations Satisfied by the Components of \mathbf{L}	276
Problem 2	Explicit Expressions for L_i and \mathbf{L}^2 in Spherical Coordinates	277
Problem 3	Orthogonality of Spherical Harmonics	279
Problem 4	Commutation Relations of a Vector Operator and \mathbf{L}	280
Problem 5	The Functions $\mathcal{Y}_{lm}(r, \Omega) = r^l Y_{lm}(\Omega)$ are Solutions of the Laplace Equation	280
Problem 6	Measurements of \mathbf{L}^2 and L_z : An Example	281
Problem 7	Measurement of \mathbf{L}^2 : An Example	283
Problem 8	Ehrenfest's Relations for $\langle \mathbf{L}(t) \rangle$	285
Problem 9	Some Identities Involving \mathbf{r} , \mathbf{p} , and \mathbf{L}	286
Problem 10	More Identities Involving $\hat{\mathbf{L}}$ for a System of N Particles	287
Problem 11	Expectation Values of $\langle \hat{L}_\alpha \rangle_{lm}$ and $\langle \hat{L}_\alpha^2 \rangle_{lm}$	289
10	Bound States in a Central Potential: Applications	291
10.1	Hydrogen-Like Atoms	291
10.2	Harmonic Oscillator in Three Dimensions	293
10.3	The Two-Body Problem	295
10.4	Problems	296
Problem 1	Solution of the Schrödinger Equation by Separation of Variables	296
Problem 2	Solution of the Radial Equation for the Hydrogen Atom	299
Problem 3	The Radial Equation for the Isotropic Harmonic Oscillator in Three Dimensions	302

Problem 4	Alternative Solution of the Isotropic Harmonic Oscillator in Three Dimensions	303
Problem 5	Differences in the Energy Spectra of Hydrogen and Deuterium	304
Problem 6	The Schrödinger Equation in Two Dimensions	305
Problem 7	S-Wave Bound States in a Spherical Potential Well	309
Problem 8	Particle Constrained in a Spherical Potential Well	312
Problem 9	Given $\psi_{lm}(\mathbf{r}) = r^\beta e^{-\gamma r} Y_{lm}(\Omega)$, Can All H-Atom Eigenfunctions be Determined?	314
Problem 10	Probability for Electron in H Atom to be in Classically Forbidden Region	314
Problem 11	Relation Between the Radial Equations of the Hydrogen Atom and the Harmonic Oscillator	315
Problem 12	WKB Approximation of the Hydrogen-like Atom Spectrum	317
Problem 13	S-Wave Particle in Central Potential $V(r) = -V_0 e^{-r/a}$	322
Problem 14	Relation Between S-Wave Functions Evaluated at the Origin and $V'(r)$	324
Problem 15	Virial Theorem in Three Dimensions	325
Problem 16	Harmonic Oscillator in Two Dimensions in Plane Polar Coordinates	327
11	Angular Momentum: General Properties	332
11.1	Raising and Lowering Operators: Definitions and Properties	332
11.2	Determining the Eigenvalues of $\hat{\mathbf{J}}^2$ and \hat{J}_3	333
11.3	Basis Consisting of Eigenstates of $\hat{\mathbf{J}}^2$ and \hat{J}_3	336
11.4	Problems	337
Problem 1	Commutation Relations of Matrices Representing the \hat{J}_i	337
Problem 2	Angular Momentum $j = 1/2$ (or Spin $1/2$)	337
Problem 3	Matrix Representation of the Angular Momentum Components	339
Problem 4	Averages of \hat{J}_x , \hat{J}_z and \hat{J}_x^2 , \hat{J}_z^2 on a State with $j = 1$	340
Problem 5	Construction of the State of a Spin-1 Particle Polarized in a General Direction	341
Problem 6	Construction of Spherical Harmonics	342
Problem 7	Energy Spectrum of an Asymmetric Rotator	344
Problem 8	Angular Momentum Algebra and the Harmonic Oscillator in Two Dimensions	345
Problem 9	Construction of Angular Momentum Eigenstates	347
Problem 10	Observables Commuting with Two Components of $\hat{\mathbf{J}}$	349
Problem 11	Orbital Angular Momentum and Parity	349
Problem 12	Verification of the Properties Satisfied by \hat{L}_\pm when Acting on $Y_{lm}(\Omega)$	351
Problem 13	Transformation of States and Vector Operators Under Rotations	353
Problem 14	Algebraic Derivation of the Hydrogen Atom Spectrum	356
12	Spin; Charged Particle in an Electromagnetic Field	365
12.1	Treatment of a Spin-1/2 Particle	365
12.2	Charged Particle in an Electromagnetic Field	366
12.3	Charged Particle with Spin in a Uniform Magnetic Field	369
12.4	Problems	370

Problem 1	Coordinate Representation of the Operator $\hat{\mathbf{p}} \cdot \hat{\mathbf{S}}$	370
Problem 2	Spinor Wave Function of the Hydrogen Atom: An Example	371
Problem 3	The Lorentz Force in Terms of Scalar and Vector Potentials	372
Problem 4	Lagrange and Hamilton Equations of Motion for Charged Particle in EM Field	372
Problem 5	Schrödinger Equation for Charged Particle in EM Field and Gauge Invariance	373
Problem 6	Spin-1/2 Particle in a Time-dependent Magnetic Field	374
Problem 7	Scattering of Spin-1/2 Particle in Spin-dependent δ -Function Potential	376
Problem 8	Pauli Hamiltonian for Electron in EM Field	378
Problem 9	A Simplified Analysis of the Stern–Gerlach Experiment	379
Problem 10	Polarizing a Beam of Spin-1/2 Particles by a Magnetic Interaction	382
Problem 11	Time Evolution of a Spin-1 State in a Time-Dependent Magnetic Field	384
Problem 12	Neutron Interferometry and 4π Rotations of Spinor Wave Functions	386
Problem 13	Charged Spinless Particle Confined in Plane Perpendicular to Uniform Magnetic Field	388
Problem 14	Particle Confined to a Cylindrical Region	391
Problem 15	Ahronov–Bohm Effect for Charged Spinless Particle Confined in Cylindrical Shell	394
Problem 16	Electron in a Uniform Magnetic Field	397
Problem 17	Spin Precession in a Magnetic Field	404
Problem 18	Spin Precession in a Magnetic Field: Alternative Treatment	405
Problem 19	Magnetic Resonance	406

13 Addition of Angular Momenta 411

13.1	Eigenvalues of the Total Angular Momentum $\hat{\mathbf{J}}^2$	412
13.2	Clebsch–Gordan Coefficients	414
13.3	Problems	416

Problem 1	Determining the Minimum Value $ j_1 - j_2 $ in the Addition of Two Angular Momenta	416
Problem 2	Angular Momentum in the Deuterium Atom	417
Problem 3	Combining Angular Momenta 1 and 1/2	417
Problem 4	Combining the Spins of Three Spin-1/2 Particle	418
Problem 5	Projection Operators onto $j = l \pm 1/2$ States	419
Problem 6	Angular Momentum and Parity Conservation in a Two-Particle Decay	420
Problem 7	The Addition Formula for Two Spherical Harmonics	422
Problem 8	Positronium in a Magnetic Field	424
Problem 9	Decay of a Spin-1/2 Particle into a Spin-0 Particle and a Spin-1/2 Particle	426
Problem 10	Addition of Three Angular Momenta	429

14 Approximation Methods 433

14.1	Non-Degenerate Perturbation Theory	433
14.2	Degenerate Perturbation Theory	436
14.3	The Variational Method	439
14.4	Problems	441

Problem 1	Second-Order Correction to an Energy Eigenstate	441
Problem 2	Wave-Function Renormalization Constant	441
Problem 3	One-Dimensional Harmonic Oscillator in a Uniform Electric Field	442
Problem 4	Particle in an Infinite Potential Well Subject to a Barrier Perturbation	443
Problem 5	Alternative Derivation of First-Order Energy in Degenerate Perturbation Theory	447
Problem 6	Particle Confined in a Box and Subject to a Perturbation	448
Problem 7	Effect of Finite Nuclear Size on Energy Spectrum of Hydrogen-Like Atom	450
Problem 8	Hydrogen Atom in a Static External Electric Field: Stark Effect	453
Problem 9	Spin-1/2 System in a Uniform Magnetic Field in a Generic Direction	455
Problem 10	Charged Particle Constrained on a Circle in a Uniform Electric Field	457
Problem 11	Electron Interacting with Two ^3He Nuclei in a Magnetic Field	460
Problem 12	Nucleus with Spin in Non-Uniform Electric Field and Weak Magnetic Field	463
Problem 13	Degenerate Perturbation Theory in Second Order: An Example	468
Problem 14	Charged Particle in Anisotropic Harmonic Oscillator Potential and Uniform Magnetic Field	470
Problem 15	Hydrogen Atom in a Uniform Electric Field: Induced Electric Dipole Moment	473
Problem 16	A Symmetric Rotator under the Influence of a Small Perturbation	475
Problem 17	Electron in a Harmonic Potential under the Action of a Uniform Electric Field	476
Problem 18	Leading-Order Correction for the Hydrogen-Atom Ground-State Energy in an Electric Field	483
Problem 19	Perturbative Calculation of the Relativistic Kinetic Energy Term in Hydrogen-Like-Atom Levels	488
Problem 20	Derivation of the Brillouin–Wigner Perturbation Theory	492
Problem 21	Variational Calculation of the Ground-State Energy in a Screened Coulomb Potential	496
Problem 22	Variational Calculation of the Hydrogen Atom Ground-State Energy	497
Problem 23	Variational Calculation of the Helium Atom Ground-State Energy	498
Problem 24	The Born–Oppenheimer Approximation	500
Problem 25	Variational Calculation of the H_2^+ Molecular Ion Binding Energy	506
Problem 26	Estimating Bound-State Energies of a Hamiltonian with the Variational Method	511

15 Scattering by a Potential	516
15.1 Cross Section and Scattering Wave Function	516
15.2 Integral Equation for Scattering, Lippmann–Schwinger Equation, and Born Approximation	519
15.3 Scattering by a Central Potential: Phase-Shift Method	522
15.4 Problems	527
Problem 1 Verifying That the Asymptotic Wave Function Satisfies the Schrödinger Equation	527

Problem 2	Born Approximation for Scattering in Yukawa and Coulomb Potentials	527
Problem 3	Born Approximation for Scattering in a Gaussian Potential	528
Problem 4	Alternative Derivation of the PDE Satisfied by the Scattering Green's Function	529
Problem 5	On the Validity of the Born Approximation	530
Problem 6	Optical Theorem	531
Problem 7	Integral Equation for Bound States	533
Problem 8	Derivation of the Scattering and Bound-State Green's Functions	534
Problem 9	Perturbative Expansion of the Scattering Wave Function and Scattering Amplitude	536
Problem 10	Relating the Partial Wave Amplitude f_{kl} to the Phase Shift δ_{kl}	537
Problem 11	Low-Energy Behavior of the Phase Shifts	538
Problem 12	Hard-Sphere Potential: Phase Shifts, Low- and High-Energy Limits of Total Cross Section	539
Problem 13	Study of the Spherical Potential Well in S -Waves; Resonances	541
Problem 14	S -Wave Bound- and Scattering-State Problem in an Attractive δ -Shell Potential	547
Problem 15	Scattering in a Repulsive δ -Shell Potential	548
Problem 16	Integral Equation for Scattering in One Dimension	552
Problem 17	Partial Wave Expansion of the Free-Particle Wave Function	554
Problem 18	Derivation of an Integral Relation for the Phase Shift	557
Problem 19	Partial Wave Expansion of Integral Equation for Scattering in Central Potential	558
Problem 20	Phase Shift as Integral over Radial Scattering Solution of Schrödinger Equation	564
Problem 21	Scattering in a Spin-Dependent Potential	566
Problem 22	Phase Shift in the Born Approximation	570
Problem 23	Effective Range Theory	572
Problem 24	Phase Shift in the High-Energy Approximation	574
Problem 25	Eikonal Approximation for the Scattering Amplitude	578

16 Symmetry Transformations of States and Operators 582

16.1	Space and Time Translations	584
16.2	Rotations	585
16.3	Discrete Symmetries: Space Inversion and Time Reversal	588
16.3.1	Time Reversal; Properties of Anti-Unitary Operators	588
16.3.2	Transformation of States and Operators under Time Reversal	590
16.4	Problems	593
Problem 1	Transformation of the Position Operator under a Translation	593
Problem 2	Charged Particle in a Harmonic Oscillator Potential and a Uniform Electric Field	593
Problem 3	Periodic Potential and Bloch Waves	594
Problem 4	Periodic Potential and Bloch Waves: An Alternative Treatment	596
Problem 5	The Kronig–Penney Model	597
Problem 6	Construction of the Rotation Matrices from the Infinitesimal Generators	600

Problem 7	Explicit Expression for the Rotation Operator in Spin Space	601
Problem 8	Rotation by 2π	602
Problem 9	Consequences of Rotational Invariance	603
Problem 10	Matrix Elements of Vector Operators and Rotations	604
Problem 11	Transformation of a Spin-1/2 Angular Momentum Operator Under a General Rotation	606
Problem 12	Explicit Derivation of the Transformation Properties of $\hat{\mathbf{r}}$, $\hat{\mathbf{p}}$, and $\hat{\mathbf{L}}$ under Rotations	607
Problem 13	Construction of a Spin State Polarized Along a Generic Direction $\hat{\mathbf{n}}$	608
Problem 14	Construction of the Unitary Operator Inducing Galilean Transformations	609
Problem 15	Consequences of Space Inversion Symmetry	611
Problem 16	A Model for the Ammonia Molecule and Broken Parity Symmetry	612
Problem 17	Unitarity Implies Linearity and Anti-Unitarity Implies Antilinearity	616
Problem 18	The Unitary Operator $\hat{U}_{\mathcal{T}}$ in $\hat{\Omega}_{\mathcal{T}} = \hat{U}_{\mathcal{T}} \hat{K}$ Depends on the Representation	617
Problem 19	Time Evolution of a State and of Its Time-Reversed Partner	617
Problem 20	On Eigenfunctions of Non-Degenerate Eigenvalues of a Time-Reversal-Invariant Observable	618
Problem 21	Transformation of States under $\hat{\Omega}_{\mathcal{T}}^2$; Kramers' Degeneracy	618
Problem 22	Time-Reversal Invariance and Scattering of Spinless Particles	619

17	Rotation Matrices and the Wigner–Eckart Theorem; Fine and Hyperfine Structure of Energy Levels in Hydrogen-Like Atoms	622
17.1	Irreducible Tensor Operators and the Wigner–Eckart Theorem	623
17.2	Matrix Elements of Scalar and Vector Operators	626
17.3	Relativistic Corrections to Hydrogen-Like-Atom Hamiltonian	627
17.4	Problems	631
Problem 1	Some Properties of Rotation Matrices	631
Problem 2	Physical Interpretation of the Matrix Elements $D_{m,m'}^{(j)}(\alpha, \beta, \gamma)$	632
Problem 3	Explicit Calculation of the Transform of an Angular Momentum Component under Rotation	632
Problem 4	Construction of the Rotation Matrix $\underline{D}^{(1)}(\alpha, \beta, \gamma)$	633
Problem 5	Commutation Relations of $\hat{\mathbf{J}}$ with the Spherical Components of a Vector Operator	635
Problem 6	Combining Two ITOs	635
Problem 7	ITOs of Ranks 0, 1, and 2 from Two Vector Operators	636
Problem 8	Product of Two Rotation Matrices	638
Problem 9	Transformation Law of ITOs under Rotations	639
Problem 10	Transformation Law under Rotations of ITOs of Rank 1 (Vector Operators)	639
Problem 11	The Rotation Matrices as Polynomials of Order $2j$ in $\sin(\beta/2)$ and $\cos(\beta/2)$	641
Problem 12	Some Properties of the Rotation Matrices	642
Problem 13	An Explicit Derivation of the Rotation Matrices	645

Problem 14	Rotating the States or Rotating the Operator: An Example	648
Problem 15	Fine Structure of the Hydrogen Atom	651
Problem 16	Spin–Orbit Corrections to the Ground and First-Excited Levels of the Hydrogen Atom	655
Problem 17	Effects of Relativistic Corrections on 1s, 2s, and 2p Levels of the Hydrogen Atom	658
Problem 18	Hydrogen Atom in a Magnetic Field (Zeeman and Paschen–Back Effects)	659
Problem 19	Hyperfine Structure of the Hydrogen Atom	661
Problem 20	The Tensor Term in the Hyperfine Interaction	664
Problem 21	ITOs and Time-Reversal Invariance	667
18	Time-Dependent Perturbation Theory	669
18.1	Perturbative Expansion for the Time Evolution Operator	669
18.2	Special Cases: Constant and Periodic Perturbations	671
18.3	Problems	673
Problem 1	Time Evolution in a Two-State System	673
Problem 2	Spin-1 System Perturbed by an Oscillating Field	676
Problem 3	Alternative Derivation of Time-Dependent Perturbation Theory	678
Problem 4	Elastic Scattering Cross Section in Born Approximation from Fermi’s Golden Rule	680
Problem 5	Positronium in Static and Oscillating Magnetic Fields	681
Problem 6	The Transition $i \rightarrow i$	686
Problem 7	Hydrogen Atom in Time-Dependent EM Fields: Doppler Effect and Recoil Energy	689
Problem 8	Two Spin-1/2 Particles of Opposite Charge in a Time-Dependent Magnetic Field	692
Problem 9	Hydrogen Atom in a Time-Dependent Electric Field	694
Problem 10	Charged Harmonic Oscillator Subjected to an Electric Field Pulse	697
Problem 11	A One-Dimensional Model for the Photoelectric Effect	701
Problem 12	Inelastic Scattering of a Projectile at a Target: A Simple Model	703
Problem 13	Ionization of a Hydrogen Atom by an External Electromagnetic Field	705
Problem 14	Cross Sections for Stimulated Absorption and Emission in Hydrogen	710
Problem 15	Spontaneous Emission: Selection Rules	712
Problem 16	The $2p \rightarrow 1s$ Transition in Hydrogen	713
Problem 17	Theory of the Line Width	715
Problem 18	Formal Scattering Theory	720
19	Systems of Identical Particles	726
19.1	States of Identical Particles	727
19.2	Problems	729
Problem 1	Energy Levels of Three Identical Fermions or Three Identical Bosons	729
Problem 2	When Can the Symmetrization Postulate Be Ignored?	730
Problem 3	Properties of the Exchange Operator	732
Problem 4	Two Bosons or Two Fermions in a Central Potential	733

Problem 5	Two-Particle Transition Amplitudes in a Central Potential	735
Problem 6	Properties of Permutation Operators for Three Particles	739
Problem 7	Symmetrizer and Antisymmetrizer and N -Particle States	741
Problem 8	Expectation Value of a Totally Symmetric Operator on an N -Particle Antisymmetric State	744
Problem 9	The Fermi Gas	745
Problem 10	White Dwarf Stars	748
Problem 11	The Thomas–Fermi Approximation for Many-Electron Atoms	751

<i>Bibliography</i>		755
<i>Index</i>		756