

Metric Spaces

A Companion to Analysis

This textbook presents the theory of *Metric Spaces* necessary for studying analysis beyond one real variable. Rich in examples, exercises and motivation, it provides a careful and clear exposition at a pace appropriate to the material.

The book covers the main topics of metric space theory that the student of analysis is likely to need. Starting with an overview defining the principal examples of metric spaces in analysis (chapter 1), it turns to the basic theory (chapter 2) covering open and closed sets, convergence, completeness and continuity (including a treatment of continuous linear mappings). There is also a brief dive into general topology, showing how metric spaces fit into a wider theory. The following chapter is devoted to proving the completeness of the classical spaces. The text then embarks on a study of spaces with important special properties. Compact spaces, separable spaces, complete spaces and connected spaces each have a chapter devoted to them. A particular feature of the book is the occasional excursion into analysis. Examples include the Mazur–Ulam theorem, Picard’s theorem on existence of solutions to ordinary differential equations, and space filling curves.

This text will be useful to all undergraduate students of mathematics, especially those who require metric space concepts for topics such as multivariate analysis, differential equations, complex analysis, functional analysis, and topology. It includes a large number of exercises, varying from routine to challenging. The prerequisites are a first course in real analysis of one real variable, an acquaintance with set theory, and some experience with rigorous proofs.

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