

# Contents

<i>Preface</i>	<i>vii</i>
<i>0 Prologue</i>	<i>1</i>
0.1 <i>The Language of Set Theory</i>	<i>1</i>
0.2 <i>Orderings</i>	<i>4</i>
0.3 <i>Cardinality</i>	<i>6</i>
0.4 <i>More about Well Ordered Sets</i>	<i>9</i>
0.5 <i>The Extended Real Number System</i>	<i>10</i>
0.6 <i>Metric Spaces</i>	<i>13</i>
0.7 <i>Notes and References</i>	<i>16</i>
<i>1 Measures</i>	<i>19</i>
1.1 <i>Introduction</i>	<i>19</i>
1.2 <i><math>\sigma</math>-algebras</i>	<i>21</i>
1.3 <i>Measures</i>	<i>24</i>
1.4 <i>Outer Measures</i>	<i>28</i>
1.5 <i>Borel Measures on the Real Line</i>	<i>33</i>
1.6 <i>Notes and References</i>	<i>40</i>



2	<i>Integration</i>	43
2.1	<i>Measurable Functions</i>	43
2.2	<i>Integration of Nonnegative Functions</i>	49
2.3	<i>Integration of Complex Functions</i>	52
2.4	<i>Modes of Convergence</i>	60
2.5	<i>Product Measures</i>	64
2.6	<i>The <math>n</math>-dimensional Lebesgue Integral</i>	70
2.7	<i>Integration in Polar Coordinates</i>	77
2.8	<i>Notes and References</i>	81
3	<i>Signed Measures and Differentiation</i>	85
3.1	<i>Signed Measures</i>	85
3.2	<i>The Lebesgue-Radon-Nikodym Theorem</i>	88
3.3	<i>Complex Measures</i>	93
3.4	<i>Differentiation on Euclidean Space</i>	95
3.5	<i>Functions of Bounded Variation</i>	100
3.6	<i>Notes and References</i>	109
4	<i>Point Set Topology</i>	113
4.1	<i>Topological Spaces</i>	113
4.2	<i>Continuous Maps</i>	119
4.3	<i>Nets</i>	125
4.4	<i>Compact Spaces</i>	128
4.5	<i>Locally Compact Hausdorff Spaces</i>	131
4.6	<i>Two Compactness Theorems</i>	136
4.7	<i>The Stone-Weierstrass Theorem</i>	138
4.8	<i>Embeddings in Cubes</i>	143
4.9	<i>Notes and References</i>	146
5	<i>Elements of Functional Analysis</i>	151
5.1	<i>Normed Vector Spaces</i>	151
5.2	<i>Linear Functionals</i>	157
5.3	<i>The Baire Category Theorem and its Consequences</i>	161
5.4	<i>Topological Vector Spaces</i>	165
5.5	<i>Hilbert Spaces</i>	171
5.6	<i>Notes and References</i>	179



6	<i><math>L^p</math> Spaces</i>	181
6.1	<i>Basic Theory of <math>L^p</math> Spaces</i>	181
6.2	<i>The Dual of <math>L^p</math></i>	188
6.3	<i>Some Useful Inequalities</i>	193
6.4	<i>Distribution Functions and Weak <math>L^p</math></i>	197
6.5	<i>Interpolation of <math>L^p</math> Spaces</i>	200
6.6	<i>Notes and References</i>	208
7	<i>Radon Measures</i>	211
7.1	<i>Positive Linear Functionals on <math>C_c(X)</math></i>	211
7.2	<i>Regularity and Approximation Theorems</i>	216
7.3	<i>The Dual of <math>C_0(X)</math></i>	221
7.4	<i>Products of Radon Measures</i>	226
7.5	<i>Notes and References</i>	231
8	<i>Elements of Fourier Analysis</i>	235
8.1	<i>Preliminaries</i>	235
8.2	<i>Convolutions</i>	239
8.3	<i>The Fourier Transform</i>	247
8.4	<i>Summation of Fourier Integrals and Series</i>	257
8.5	<i>Pointwise Convergence of Fourier Series</i>	263
8.6	<i>Fourier Analysis of Measures</i>	270
8.7	<i>Applications to Partial Differential Equations</i>	273
8.8	<i>Notes and References</i>	278
9	<i>Elements of Distribution Theory</i>	281
9.1	<i>Distributions</i>	281
9.2	<i>Compactly Supported, Tempered, and Periodic Distributions</i>	291
9.3	<i>Sobolev Spaces</i>	301
9.4	<i>Notes and References</i>	310
10	<i>Topics in Probability Theory</i>	313
10.1	<i>Basic Concepts</i>	313
10.2	<i>The Law of Large Numbers</i>	320
10.3	<i>The Central Limit Theorem</i>	325
10.4	<i>Construction of Sample Spaces</i>	328
10.5	<i>The Wiener Process</i>	330
10.6	<i>Notes and References</i>	336



<b>11 More Measures and Integrals</b>	<b>339</b>
11.1 Topological Groups and Haar Measure	339
11.2 Hausdorff Measure	348
11.3 Self-similarity and Hausdorff Dimension	355
11.4 Integration on Manifolds	361
11.5 Notes and References	363
<b>Bibliography</b>	<b>365</b>
<b>Index of Notation</b>	<b>377</b>
<b>Index</b>	<b>379</b>