

CONTENTS

List of Examples xiii

Preface xvii

1	Introduction	1
2	Linear Spaces and Transformations	3
2.1	Properties of Linear Spaces	3
2.2	Vector Spaces	4
2.3	Linear Transformations	4
3	Differentiable Manifolds	7
3.1	Charts and Coordinates	7
3.2	Definition	10
3.3	Local Coordinate Changes	11
3.4	Functions on Manifolds	13
3.5	Orientable Manifolds	13
4	Vectors and One-Forms	15
4.1	Vectors	15
4.1.1	Vectors as Tangents to Curves	16
4.1.2	Bases and Coordinates	17

4.1.3	Vector Field	19
4.1.4	Transformations	20
4.2	One-Forms	23
4.2.1	Duality	23
4.2.2	Bases	24
4.2.3	Transformations	26
4.3	Alternative Perspective	28
4.4	Lie Bracket	29
5	Tensors	33
5.1	Definition	33
5.2	Operations on Tensors	35
5.2.1	Addition	35
5.2.2	Tensor Product	35
5.2.3	Contraction	36
5.2.4	Transpose of (2,0) and (0,2) Tensors	38
5.2.5	Transpose of a (1,1) Tensor	40
5.3	Transformations	41
5.3.1	Tetrad Formalism	43
5.3.2	Pseudotensors	46
5.4	Kronecker or Identity Tensor	46
5.5	Logarithms and Exponentials of (1,1) Tensors	46
5.6	Tensor Densities and Capacities	47
5.6.1	Pseudotensor Densities and Capacities	48
5.7	Levi-Civita Density and Capacity	48
5.8	Determinant of Rank-2 Tensors	50
5.9	Inverse of Rank-2 Tensors	50
5.10	Metric Tensor	51
5.10.1	Formulation	51
5.10.2	Geometrical Meaning	55
5.10.3	Norm of Vectors and One-Forms	56
5.10.4	Metric in Tetrads	56
5.11	Adjoint of a (1,1) Tensor	57

5.12	Tensor Densities and Capacities Revisited	58
5.13	Levi-Civita Pseudotensor	58
5.14	Kronecker Determinants	60
5.15	Rotations	63
5.15.1	Euler Angles	64
5.15.2	Rodrigues's Formula	66
6	Maps between Manifolds	71
6.1	Maps	71
6.2	Maps between Manifolds of Different Dimensions	72
6.2.1	Pullback	72
6.2.2	Pushforward	74
6.3	Maps between Manifolds of the Same Dimensions	76
7	Differentiation on Manifolds	85
7.1	Covariant Derivative	85
7.1.1	Formulation	85
7.1.2	Transformation of Connection Coefficients	89
7.1.3	Divergence	91
7.1.4	Parallel Transport	91
7.1.5	Torsion and Curvature Tensors	94
7.1.6	Bianchi Identities	97
7.1.7	Torsion-Free Connection	97
7.1.8	Covariant Derivative of the Metric Tensor	98
7.1.9	Mixed Covariant Derivative in Tetrad Basis	101
7.1.10	Spin Connection	102
7.1.11	Contracted Bianchi Identities	102
7.1.12	Covariant Derivative of Tensor Densities and Capacities	107
7.1.13	Nonmetricity	107
7.2	Euler Derivative	112
7.3	Lie Derivative	114
7.3.1	Lie Derivative of Vectors	114
7.3.2	Geometrical Interpretation	117

7.3.3	Autonomous Lie Derivative	118
7.3.4	Lie Derivative of One-Forms	118
7.3.5	Lie Derivative of (p, q) Tensors	119
7.3.6	Lie Derivative of Functions	120
7.3.7	Lie Derivative of Metric Tensors	120
7.3.8	Lie Derivative of Levi-Civita Tensor	123
8	Differential Forms	127
8.1	Definition	127
8.2	Operations on Forms	130
8.2.1	Addition	130
8.2.2	Exterior Product	130
8.2.3	Interior Product	133
8.3	k -Vectors	133
8.4	Hodge Dual	134
8.5	Volumes	135
8.5.1	Properties	136
8.6	Surfaces	137
8.7	Exterior Derivative	139
8.7.1	Coordinate-Free Definition	140
8.7.2	Exact Forms	142
8.7.3	Commutativity with Pullback and Pushforward	144
8.8	Lie Derivative of a Form	148
8.9	Vector- and Tensor-Valued Forms	150
8.9.1	Transformations of Tensor-Valued Forms	151
8.9.2	Operations on Tensor-Valued Forms	159
8.9.3	Connection One-Forms	161
8.9.4	Torsion Two-Forms	162
8.9.5	Exterior Covariant Derivative	164
8.9.6	Covariant Lie Derivative	167
8.9.7	Curvature Two-Forms	169
8.9.8	Commutator of Covariant Lie and Exterior Covariant Derivatives	171

8.9.9	Bianchi Identities Revisited.....	172
8.9.10	Nonmetricity Revisited	173
8.10	Integration of Forms	174
8.10.1	Line Integrals.....	174
8.10.2	Surface Integrals	175
8.10.3	Volume Integrals.....	177
8.11	Stokes's Theorem	178
8.11.1	Fundamental Theorem of Calculus	178
8.11.2	Green's Theorem	179
8.11.3	Gauss's Theorem	180
8.11.4	Stokes's Theorem.....	180
8.11.5	Variational Principles	180
8.11.6	Noether's Theorem	181
8.11.7	Applications	183

Glossary 211

Bibliography 219

Author Index 223

General Index 225