

Contents

1	Notation, Definitions, and Basic Concepts	1
1.1	Notation	1
1.1.1	Algebra	1
1.1.2	Functions and Differential and Integral Operations	3
1.1.3	Spaces of Functions	5
1.2	Mathematical Tools	8
1.3	Moving Domain	19
2	Equations of Motion	27
2.1	Barotropic Fluid	27
2.1.1	Initial Data	27
2.1.2	Boundary Conditions	28
2.1.3	Continuity Equation	29
2.1.4	Momentum Equation	32
2.1.5	Remark on Test Functions	33
2.1.6	Energy Inequality	33
2.1.7	Weak Solutions	37
2.2	Heat Conducting Fluid	39
2.2.1	Initial Data	39
2.2.2	Boundary Conditions	40
2.2.3	Continuity Equation	40
2.2.4	Momentum Equation	40
2.2.5	Total Energy Balance	41
2.2.6	Entropy Balance	43
2.2.7	Hypothesis	44
2.2.8	Variational Solutions	46
3	Barotropic Viscous Fluid with the Dirichlet Boundary Conditions	51
3.1	Existence Theorem and Ideas of the Proof	52
3.2	Approximation	52
3.3	Modified Energy Inequality and Uniform Bounds	55
3.3.1	Energy Estimates	55

3.3.2	Pressure Estimates	57
3.4	Limit as $\varepsilon \rightarrow 0$	61
3.4.1	Limit Equations	61
3.4.2	Pointwise Convergence of the Density	63
3.5	Concluding Remarks	70
4	Barotropic Viscous Fluid with Slip Boundary Condition	71
4.1	Existence Theorem and Ideas of the Proof	72
4.2	Approximation	73
4.3	Modified Energy Inequality and Uniform Bounds	77
4.3.1	Energy Estimates	77
4.3.2	Pressure Estimates	79
4.4	Limit $\varepsilon \rightarrow 0$	82
4.4.1	Limit Equations	82
4.4.2	Pointwise Convergence of the Density	86
4.5	Limit $\omega \rightarrow 0$	89
4.5.1	Uniform Estimates	90
4.5.2	Limit Equations	91
4.6	Limit $\delta \rightarrow 0$	93
4.6.1	Uniform Estimates	94
4.6.2	Limit Equations	94
4.7	Finite Energy Weak Solutions for the Dirichlet Boundary Condition	99
5	Weak-Strong Uniqueness	101
5.1	The Relative Energy Inequality	102
5.2	Weak-Strong Uniqueness	107
6	Existence of Strong Solutions via Energy Methods	115
6.1	Discussion and Main Results	115
6.2	Iterative Scheme	117
6.3	Linear Continuity Equation	118
6.4	Linear Momentum Equation	126
6.4.1	Lagrangian coordinates determined by $\mathbf{V}$	127
6.4.2	Equivalence of Norms in Lagrangian and Eulerian Coordinates	129
6.4.3	Transformation of the Linear Momentum Equation	130
6.4.4	Solution of the Linear Problem in a Fixed Domain	133
6.4.5	Extension of the Boundary Data	138
6.4.6	Proof of Proposition 6.4.1	144
6.5	Convergence of the Sequence of Approximations: Proof of Theorem 6.1.1	148
7	Existence of Strong Solutions in the $L^p - L^q$ Framework	157
7.1	Main Results	157
7.2	Transformation Determined by the Solution	160

7.3	Linear Theory	166
7.3.1	$L^p - L^q$ maximal regularity	166
7.3.2	Exponential Decay	168
7.4	Proof of Theorem 7.1.1	169
7.4.1	Linearization	169
7.4.2	Nonlinear Estimates	171
7.4.3	Fixed Point Argument	174
7.5	Proof of Theorem 7.1.2	178
7.5.1	Linearization	178
7.5.2	Nonlinear Estimates	179
7.5.3	Extension of Local Solution	182
8	The Full System	185
8.1	Existence Theorem and Ideas of the Proof.....	186
8.2	Approximation	187
8.3	Existence Result for the Approximation Problem	192
8.4	Modified Energy Inequality and Uniform Bounds	204
8.4.1	Energy Estimates	204
8.4.2	Pressure Estimates	213
8.5	Limit as $\varepsilon \rightarrow 0$	216
8.5.1	Limit Equations	216
8.5.2	Pointwise Convergence of the Temperature	221
8.5.3	Pointwise Convergence of the Density	223
8.5.4	Limit Equations Revisited	227
8.6	Limit $\omega \rightarrow 0$	229
8.6.1	Uniform Estimates	230
8.6.2	Limit Equations	234
8.7	Limit as $\delta \rightarrow 0$	240
8.7.1	Uniform Estimates	240
8.7.2	Limit Equations	242
8.7.3	Pointwise Convergence of the Density	245
8.7.4	Limit Equations Revisited	251
8.8	Variational Solutions for Dirichlet Boundary Condition for the Velocity	252
	References.....	255
	Index.....	259