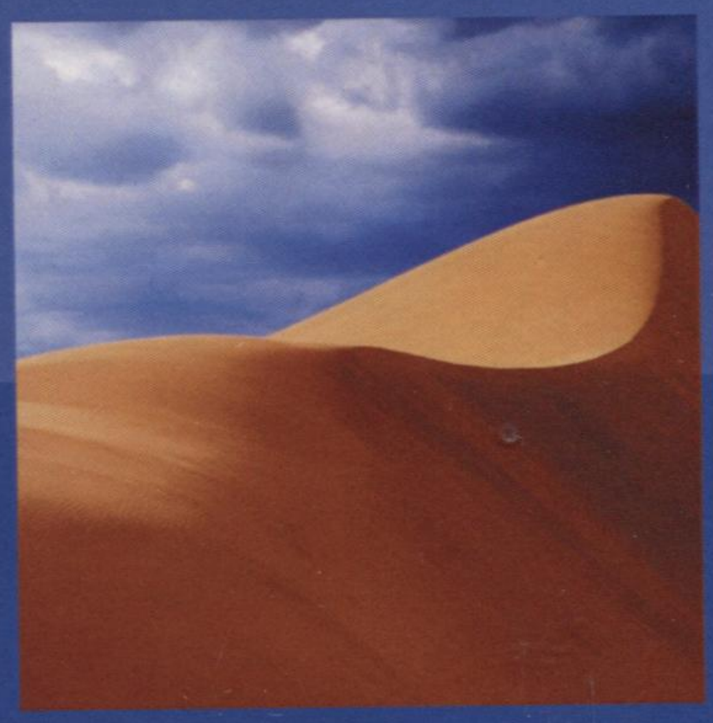
While many scientists are familiar with fractals, fewer are cognizant of the concepts of scale-invariance and universality which underlie the ubiquity of such fascinating shapes. These inherent properties emerge from the collective behavior of simple fundamental constituents. The initial chapters smoothly connect the particulate perspective developed in the companion volume, *Statistical Physics of Particles*, to the coarse-grained statistical fields studied in this textbook. It carefully demonstrates how such theories are constructed from basic principles such as symmetry and locality, and studied by innovative methods like the renormalization group. Perturbation theory, exact solutions, renormalization, and other tools are employed to demonstrate the emergence of scale invariance and universality. The book concludes with chapters related to the research of the author on non-equilibrium dynamics of interfaces, and directed paths in random media.



Covering the more advanced applications of statistical mechanics, this textbook is ideal for advanced graduate students in physics. It is based on lectures for a course in statistical physics taught by Professor Kardar at Massachusetts Institute of Technology (MIT). The large number of integrated problems introduce the reader to novel applications such as percolation and roughening. The selected solutions at the end of the book are ideal for self-study and honing calculation methods. Additional solutions are available to lecturers on a password-protected website at www.cambridge.org/9780521873413.

In this much-needed modern text, Kardar presents a remarkably clear view of statistical mechanics as a whole, revealing the relationships between different parts of this diverse subject. In two volumes, the classical beginnings of thermodynamics are connected smoothly to a thoroughly modern view of fluctuation effects, stochastic dynamics, and renormalization and scaling theory. Students will appreciate the precision and clarity in which difficult concepts are presented in generality and by example. I particularly like the wealth of interesting and instructive problems inspired by diverse phenomena throughout physics (and beyond!), which illustrate the power and broad applicability of statistical mechanics.
Leon Balents, Department of Physics, University of California, Santa Barbara

. . . *Statistical Physics of Particles* is the welcome result of an innovative and popular graduate course Kardar has been teaching at MIT for almost 20 years. It is a masterful account of the essentials of a subject which played a vital role in the development of twentieth-century physics . . . *Statistical Physics of Fields* builds on the foundation laid by the *Statistical Physics of Particles*, with an account of the revolutionary developments of the past 35 years, many of which were facilitated by renormalization group ideas. Much of the subject matter is inspired by problems in condensed matter physics, with a number of pioneering contributions originally due to Kardar himself.
David R. Nelson, Arthur K. Solomon Professor of Biophysics, Harvard University

If Landau and Lifshitz were to prepare a new edition of their classic statistical physics text they might produce a book not unlike this gem by Mehran Kardar. Indeed, Kardar is an extremely rare scientist, being both brilliant in formalism and an astoundingly careful and thorough teacher. He demonstrates both aspects of his range of talents in this pair of books, which belong on the bookshelf of every serious student of theoretical statistical physics.
H. Eugene Stanley, Director, Center for Polymer Studies, Boston University

This is one of the most valuable textbooks I have seen in a long time. Written by a leader in the field, it provides a crystal clear, elegant and comprehensive coverage of the field of statistical physics. I'm sure this book will become *the* reference for the next generation of researchers, students and practitioners in statistical physics. I wish I had this book when I was a student, but I will have the privilege to rely on it for my teaching.
Alessandro Vespignani, Center for Biocomplexity, Indiana University

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