

An anomaly is the failure of classical symmetry to survive the process of quantization and regularization. The study of anomalies is the key to a deeper understanding of quantum field theory and has played an increasingly important role in the theory over the last 20 years. This text presents all the different aspects of the study of anomalies in an accessible and self-contained way. Much emphasis is now being placed on the formulation of the theory using mathematical ideas of differential geometry and topology. This approach is followed here, and the derivations and calculations are given explicitly as an aid to students.

The comprehensive overview of the theory presented in this book will be useful to both students and researchers.

Professor Bertlmann is at the Institute for Theoretical Physics, University of Vienna.

From reviews of the hardback edition

It is a leisurely and scholarly treatment of the whole subject from the ground up.... Besides being comprehensive, the book manages to convey the depth and breadth of this profound subject as well as its links to mathematics. It is bound to become a standard reference on this fascinating topic.

R. Shankar, *Physics Today*

I am not aware of another book which deals with this ambitious topic in as thorough and up-to-date a fashion as this present volume. I recommend it without reservation both as an introduction and as a reference for experts.

H. Nicolai, *Physikalische Blätter*

Modern differential geometry and functional analysis have forged the tools for a deeper understanding in theoretical physics. Bertlmann's book does not only give the mathematical background but also illustrates these notions by subtle features of modern quantum field theory.

Professor W. Thirring, *Erwin-Schrödinger-Institut, Vienna*

... an excellent guide to many aspects of anomalies. Professor Bertlmann surveys the basis for the unexpected effects and he puts forward evidence that not only the theoretical physicist but also Nature knows and uses the anomalous symmetry breaking mechanism. Moreover, he presents the relevant concepts of geometry and topology in a detailed and explicit fashion, thereby providing a most useful and practical introduction to these topics in contemporary mathematics.

Professor R. Jackiw, *MIT*

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