

INSTITUTE OF PHYSICS

**SERIES IN HIGH ENERGY PHYSICS,
COSMOLOGY AND GRAVITATION**

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This series of books covers all aspects of theoretical and experimental high energy physics, cosmology and gravitation, and the interfaces between them. In recent years the fields of particle physics and astrophysics have become increasingly interdependent. The aim of this Series is to provide a library of books to meet the needs of students and researchers in these fields.

Branes are solitonic configurations of a string theory, which are represented by extended objects in (a higher-dimensional) spacetime. They are essential for a comprehension of the non-perturbative aspects of string theory, in particular in connection with string dualities. From the mathematical viewpoint, they are related to several important theories, such as homological mirror symmetry and quantum cohomology.

This book, based on lectures given to postgraduate students, provides an introduction to current research in some of these different areas, both in physics and mathematics. The book opens with a lucid introduction to the basic aspects of branes in string theory. Topics covered in subsequent chapters include tachyon condensation, the geometry surrounding the Gopakumar–Vafa conjecture (a duality between the $SU(N)$ Chern–Simons theory on S^3 and a IIA string theory compactified on a Calabi–Yau 3-fold), two-dimensional conformal field theory on open and unoriented surfaces, and the development of homology theory naturally attached to the deformations of vector bundles which should be relevant to the study of homological mirror symmetry.

The book will be useful for graduate students and for researchers in mathematics and theoretical physics who want to enter this exciting and challenging field.

ISBN 0-7503-0863-X



9 780750 308632 >

Institute of Physics Publishing
Bristol and Philadelphia
www.iop.org

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