

generatingfunctionology

THIRD EDITION

Herbert S. Wilf

This is the third edition of the highly successful introduction to the use of generating functions and series in combinatorial mathematics. This new edition provides a clear, unified introduction to the basic enumerative applications of generating functions.

Special Features:

- Provides applications to the cycle index of the symmetric group, permutations and square roots, counting polyominoes, and exact covering sequences
- Features an appendix on using Maple[®] and Mathematica[®] to work with generating functions
- Includes new exercises with complete solutions at the end of each chapter



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Preface	ix
1 Introductory Ideas and Examples	1
1.1 An Easy Two Term Recurrence	4
1.2 A Slightly Harder Two Term Recurrence	5
1.3 A Three Term Recurrence	9
1.4 A Three Term Boundary Value Problem	11
1.5 Two Independent Variables	15
1.6 Another 2-Variable Case	18
1.7 Exercises	26
2 Series	31
2.1 Formal Power Series	31
2.2 The Calculus of Formal Ordinary Power Series Generating Functions	35
2.3 The Calculus of Formal Exponential Generating Functions .	41
2.4 Power Series, Analytic Theory	48
2.5 Some Useful Power Series	55
2.6 Dirichlet Series, Formal theory	59
2.7 Exercises	68
3 Cards, Decks, and Hands: The Exponential Formula	77
3.1 Introduction	77
3.2 Definitions and a Question	79
3.3 Examples of Exponential Families	80
3.4 The Main Counting Theorems	83
3.5 Permutations and Their Cycles	87
3.6 Set Partitions	88
3.7 A Subclass of Permutations	89
3.8 Involutions, etc.	90

3.9	2-Regular Graphs	91
3.10	Counting Connected Graphs	92
3.11	Counting Labeled Bipartite Graphs	93
3.12	Counting Labeled Trees	95
3.13	Exponential Families and Polynomials of ‘Binomial Type.’ .	97
3.14	Unlabeled Cards and Hands	98
3.15	The Money Changing Problem	102
3.16	Partitions of Integers	107
3.17	Rooted Trees and Forests	109
3.18	Historical Notes	110
3.19	Exercises	110
4	Applications of Generating Functions	115
4.1	Generating Functions Find Averages, etc.	115
4.2	A Generatingfunctionological View of the Sieve Method . .	117
4.3	The ‘Snake Oil’ Method for Easier Combinatorial Identities	126
4.4	WZ Pairs Prove Harder Identities	138
4.5	Generating Functions and Unimodality, Convexity, etc. . . .	145
4.6	Generating Functions Prove Congruences	148
4.7	The Cycle Index of the Symmetric Group	150
4.8	How Many Permutations Have Square Roots?	155
4.9	Counting Polyominoes	159
4.10	Exact Covering Sequences	163
4.11	Waiting for a String	166
4.12	Blocks of 1	167
4.13	Exercises	170
5	Analytic and Asymptotic Methods	181
5.1	The Lagrange Inversion Formula	181
5.2	Analyticity and Asymptotics (I): Poles	185
5.3	Analyticity and Asymptotics (II): Algebraic Singularities . .	192
5.4	Analyticity and Asymptotics (III): Hayman’s Method	196
5.5	Exercises	203
A	Using Maple and Mathematica	207
A.1	Series Manipulation	208
A.2	The <code>RSolve.m</code> Routine	209
A.3	Asymptotics in Maple	211
	Solutions	213

Contents

vii

References

239

Index

243