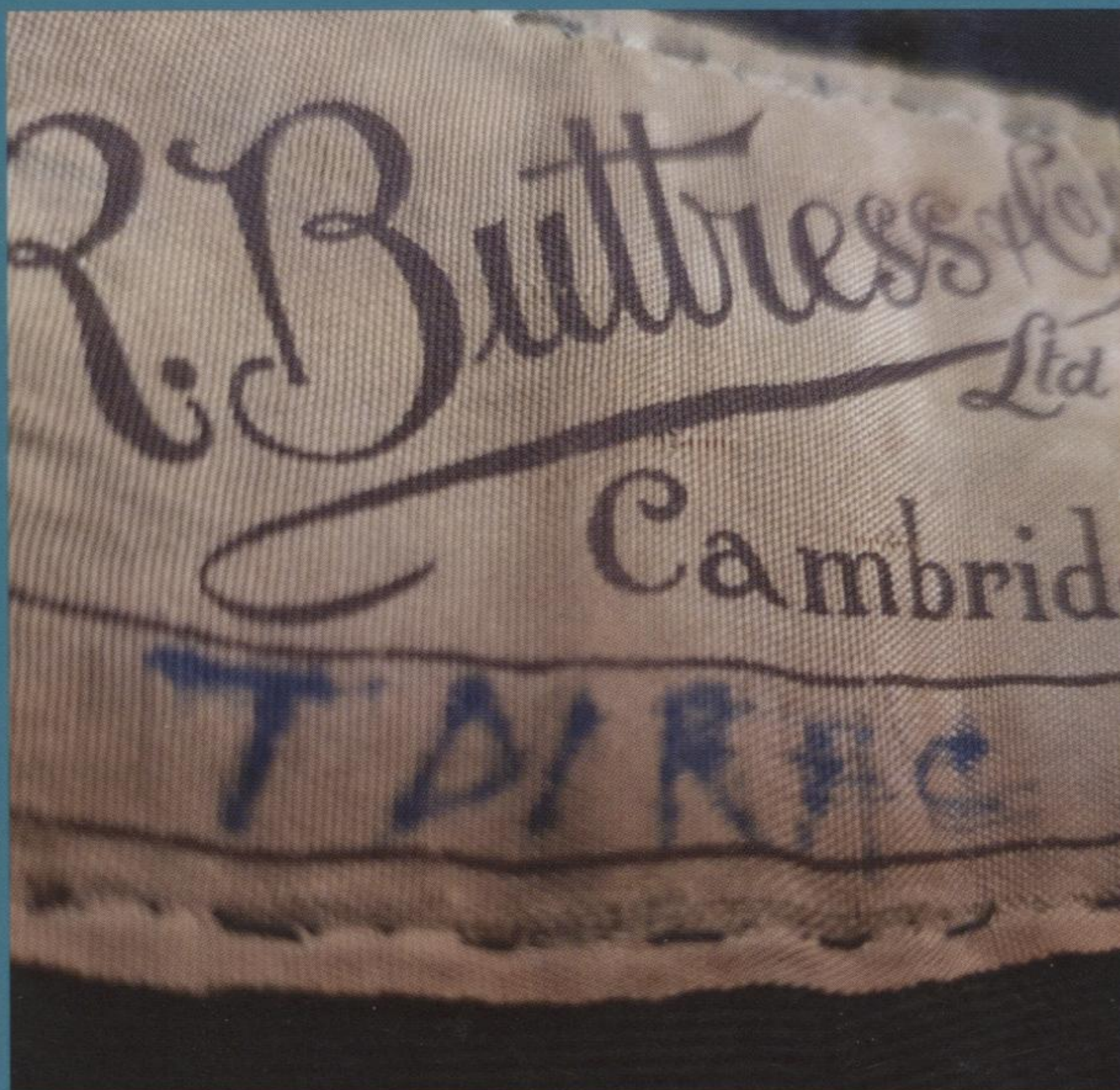


LECTURES ON THEORETICAL PHYSICS

The universe we live in is both strange and interesting. This strangeness comes about because, at the most fundamental level, the universe is governed by the laws of quantum mechanics. This is the most spectacularly accurate and powerful theory ever devised, one that has given us insights into many aspects of the world, from the structure of matter to the meaning of information. This textbook provides a comprehensive account of all things quantum. It starts by introducing the wavefunction and its interpretation as an ephemeral wave of complex probability, before delving into the mathematical formalism of quantum mechanics and exploring its diverse applications, from atomic physics and scattering, to quantum computing. Designed to be accessible, this volume is suitable for both students and researchers, beginning with the basics before progressing to more advanced topics.

DAVID TONG is a Professor of Theoretical Physics at the University of Cambridge and a Fellow of Trinity College. He is known for his contributions to quantum field theory and its application to diverse areas of physics, from condensed matter to cosmology to quantum gravity. His lecture notes on theoretical physics have gained a global following due to their clear explanations and easy-going, accessible style.



Cover Image: Front Cover: G. P. Thomson's original electron diffraction experiment. From top to bottom: didn't work, didn't work, worked. Back Cover: Dirac's misspelled name on his gown. Both photos by the author.



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