A modern introduction to Fourier analysis and selected applications which shows how these mathematical ideas can be used to study sampling theory, PDEs, probability, diffraction, musical tones, and wavelets. Ideal for students from the physical, engineering, and mathematical sciences who have mastered the usual college level courses in calculus and linear algebra.

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- Unified treatment of the analysis–synthesis equations of Fourier for periodic and aperiodic functions on R and Z, together with the Poisson identities that connect them
- A rule based calculus for finding Fourier transforms and series with minimal effort
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- A mathematically correct elementary introduction to generalized functions such as Dirac's delta and the Dirac comb, together with techniques for finding the corresponding Fourier transforms
- An emphasis on the weak limit as the preferred tool to use when working with infinite series, infinite products, and partial derivatives of generalized functions
- An introduction to Shannon's sampling theorem and its modern variations
- Fourier methods for solving the PDEs that model vibrating strings, heat flow in rods, and the diffraction of laser beams
- An efficient introduction to the orthogonal wavelets of I. Daubechies and corresponding filter banks
- Additive synthesis, FM synthesis, and spectrograms for computer generated musical tones
- More than 540 exercises that provide drill; illustrate and extend mathematical ideas from the text; provide historical perspective; foster physical insight; and stimulate the development of problem solving skills

Kammler's book invents and makes practical, a substantial upper-undergraduate level course in Fourier analysis that is appealing for many curricula. It is a joy to use as a text.

Professor O. Carruth McGehee, Louisiana State University

This book represents a profound and significant advance in the way we can teach mathematics that is both modern and meaningful.

Professor Dennis M. Healy, University of Maryland

The best thing about the book is the problem sets. In addition to the knowledge they gain about Fourier transforms, series, and applications, my students develop strong problem-solving skills that serve them well in other courses.

Professor Patrick J. Van Fleet, University of St. Thomas

All things considered, Kammler's book seems to me to be the best book on Fourier analysis at this level that I have seen.

Professor John A. Synowiec, Illinois State University

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