

"Admirably fills the need for an up-to-date textbook in this area . . . it is the only book presenting key results of the theory, including those related to black holes, quantum cosmology and the derivation of general relativity from the fundamental theory of quantum spacetime. The authors achieve a good balance of big ideas and principles with the technical details."

**Lee Smolin, Perimeter Institute for Theoretical Physics**

"This is an excellent introduction to spinfoams, an area of loop quantum gravity that draws ideas also from Regge calculus, topological field theory and group field theory. It fills an important gap in the literature offering both a pedagogical overview and a platform for further developments in a forefront area of research that is advancing rapidly."

**Abhay Ashtekar, The Pennsylvania State University**

Quantum gravity is among the most fascinating problems in physics. It modifies our understanding of time, space and matter. The recent development of the loop approach has allowed us to explore domains ranging from black hole thermodynamics to the early universe.

This book provides readers with a simple introduction to loop quantum gravity, centered on its covariant approach. It focuses on the physical and conceptual aspects of the problem and includes the background material needed to enter this lively domain of research, making it ideal for researchers and graduate students.

Topics covered include quanta of space gauge theory, classical and quantum physics without time, tetrad formalism, Holst action, lattice gauge theory, Regge calculus, ADM and Ashtekar variables, Ponzano–Regge and Turaev–Viro amplitudes, kinematics and dynamics of 4d Lorentzian quantum gravity, spectrum of area and volume, coherent states, classical limit, matter couplings, graviton propagator, spinfoam cosmology and black hole thermodynamics.

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