

This volume provides a self-contained introduction to applications of loop representations, and the related topic of knot theory, in particle physics and quantum gravity. These topics are of considerable interest because they provide a unified arena for the study of the gauge invariant quantization of Yang-Mills theories and gravity, and suggest a promising approach to the eventual unification of the four fundamental forces.

The book begins with a detailed review of loop representation theory and then describes loop representations in Maxwell theory, Yang-Mills theories as well as lattice techniques. Applications in quantum gravity are then discussed, with the following chapters considering knot theories, braid theories and extended loop representations in quantum gravity.

A final chapter assesses the current status of the theory and points out possible directions for future research. First published in 1996, this title has been reissued as an Open Access publication on Cambridge Core.



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<b>1</b>	<b>Holonomies and the group of loops</b>	<b>1</b>
1.1	Introduction	1
1.2	The group of loops	3
1.3	Infinitesimal generators of the group of loops	7
1.3.1	The loop derivative	8
1.3.2	Properties of the loop derivative	10
1.3.3	Connection derivative	17
1.3.4	Contact and functional derivatives	21
1.4	Representations of the group of loops	25
1.5	Conclusions	27
<b>2</b>	<b>Loop coordinates and the extended group of loops</b>	<b>29</b>
2.1	Introduction	29
2.2	Multitangent fields as description of loops	30
2.3	The extended group of loops	32
2.3.1	The special extended group of loops	33
2.3.2	Generators of the SeL group	36
2.4	Loop coordinates	38
2.4.1	Transverse tensor calculus	38
2.4.2	Freely specifiable loop coordinates	42
2.5	Action of the differential operators	44
2.6	Diffeomorphism invariants and knots	47
2.7	Conclusions	50
<b>3</b>	<b>The loop representation</b>	<b>52</b>
3.1	Introduction	52
3.2	Hamiltonian formulation of systems with constraints	54
3.2.1	Classical theory	54
3.2.2	Quantum theory	56
3.3	Yang–Mills theories	58



3.3.1	Canonical formulation	59
3.3.2	Quantization	61
3.4	Wilson loops	63
3.4.1	The Mandelstam identities	64
3.4.2	Reconstruction property	67
3.5	Loop representation	72
3.5.1	The loop transform	76
3.5.2	The non-canonical algebra	80
3.5.3	Wavefunctions in the loop representation	85
3.6	Conclusions	87
<b>4</b>	<b>Maxwell theory</b>	88
4.1	The Abelian group of loops	89
4.2	Classical theory	91
4.3	Fock quantization	92
4.4	Loop representation	96
4.5	Bargmann representation	103
4.5.1	The harmonic oscillator	103
4.5.2	Maxwell–Bargmann quantization in terms of loops	105
4.6	Extended loop representation	109
4.7	Conclusions	112
<b>5</b>	<b>Yang–Mills theories</b>	113
5.1	Introduction	113
5.2	Equations for the loop average in QCD	115
5.3	The loop representation	118
5.3.1	$SU(2)$ Yang–Mills theories	118
5.3.2	$SU(N)$ Yang–Mills theories	121
5.4	Wilson loops and some ideas about confinement	124
5.5	Conclusions	130
<b>6</b>	<b>Lattice techniques</b>	131
6.1	Introduction	131
6.2	Lattice gauge theories: the $Z(2)$ example	133
6.2.1	Covariant lattice theory	133
6.2.2	The transfer matrix method	136
6.2.3	Hamiltonian lattice theory	138
6.2.4	Loop representation	141
6.3	The $SU(2)$ theory	144
6.3.1	Hamiltonian lattice formulation	145
6.3.2	Loop representation in the lattice	147
6.3.3	Approximate loop techniques	150
6.4	Inclusion of fermions	156



6.5	Conclusions	159
<b>7</b>	<b>Quantum gravity</b>	<b>161</b>
7.1	Introduction	161
7.2	The traditional Hamiltonian formulation	164
7.2.1	Lagrangian formalism	164
7.2.2	The split into space and time	164
7.2.3	Constraints	168
7.2.4	Quantization	169
7.3	The new Hamiltonian formulation	171
7.3.1	Tetradic general relativity	172
7.3.2	The Palatini action	173
7.3.3	The self-dual action	174
7.3.4	The new canonical variables	175
7.4	Quantum gravity in terms of connections	179
7.4.1	Formulation	179
7.4.2	Triads to the right and the Wilson loop	180
7.4.3	Triads to the left and the Chern–Simons form	185
7.5	Conclusions	187
<b>8</b>	<b>The loop representation of quantum gravity</b>	<b>188</b>
8.1	Introduction	188
8.2	Constraints in terms of the $T$ algebra	189
8.3	Constraints via the loop transform	192
8.4	Physical states and regularization	196
8.4.1	Diffeomorphism constraint	196
8.4.2	Hamiltonian constraint: formal calculations	197
8.4.3	Hamiltonian constraint: regularized calculations	201
8.5	Conclusions	208
<b>9</b>	<b>Loop representation: further developments</b>	<b>209</b>
9.1	Introduction	209
9.2	Inclusion of matter: Weyl fermions	209
9.3	Inclusion of matter: Einstein–Maxwell and unification	215
9.4	Kalb–Ramond fields and surfaces	218
9.4.1	The Abelian group of surfaces	218
9.4.2	Kalb–Ramond fields and surface representation	220
9.5	Physical operators and weaves	221
9.5.1	Measuring the geometry of space in terms of loops	222
9.5.2	Semi-classical states: the weave	228
9.6	2+1 gravity	230
9.7	Conclusions	237



<b>10</b>	<b>Knot theory and physical states of quantum gravity</b>	<b>238</b>
10.1	Introduction	238
10.2	Knot theory	239
10.3	Knot polynomials	242
10.3.1	The Artin braid group	243
10.3.2	Skein relations, ambient and regular isotopies	245
10.3.3	Knot polynomials from representations of the braid group	249
10.3.4	Intersecting knots	251
10.4	Topological field theories and knots	253
10.4.1	Chern–Simons theory and the skein relations of the Jones polynomial	254
10.4.2	Perturbative calculation and explicit expressions for the coefficients	260
10.5	States of quantum gravity in terms of knot polynomials	264
10.5.1	The Kauffman bracket as a solution of the constraints with cosmological constant	264
10.5.2	The Jones polynomial and a state with $\Lambda = 0$	265
10.5.3	The Gauss linking number as the key to the new solution	272
10.6	Conclusions	274
<b>11</b>	<b>The extended loop representation of quantum gravity</b>	<b>275</b>
11.1	Introduction	275
11.2	Wavefunctions	277
11.3	The constraints	280
11.3.1	The diffeomorphism constraint	281
11.3.2	The Hamiltonian constraint	283
11.4	Loops as a particular case	285
11.5	Solutions of the constraints	289
11.6	Regularization	291
11.6.1	The smoothness of the extended wavefunctions	292
11.6.2	The regularization of the constraints	294
11.7	Conclusions	300
<b>12</b>	<b>Conclusions, present status and outlook</b>	<b>302</b>
12.1	Gauge theories	302
12.2	Quantum gravity	304
	<i>References</i>	309
	<i>Index</i>	319