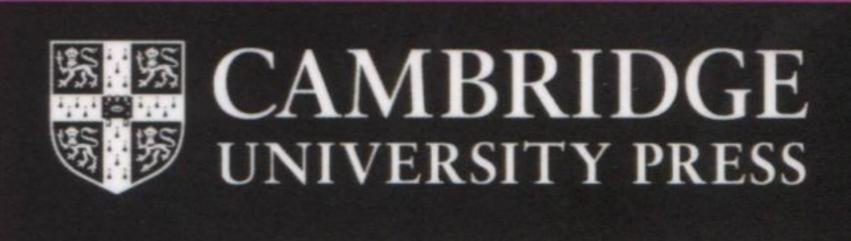
This volume provides a self-contained introduction to applications of loop representations, and the related topic of knot theory, in particle physics and quantum gravity. These topics are of considerable interest because they provide a unified arena for the study of the gauge invariant quantization of Yang-Mills theories and gravity, and suggest a promising approach to the eventual unification of the four fundamental forces. The book begins with a detailed review of loop representation theory and then describes loop representations in Maxwell theory, Yang-Mills theories as well as lattice techniques. Applications in quantum gravity are then discussed, with the following chapters considering knot theories, braid theories and extended loop representations in quantum gravity. A final chapter assesses the current status of the theory and points out possible directions for future research. First published in 1996, this title has been reissued as an Open Access publication on Cambridge Core.



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