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**Spectral Theory of Canonical Differential Systems. Method of
Operator Identities**

Theorems of factorising matrix functions and the operator identity method play an essential role in this book in constructing the spectral theory (direct and inverse problems) of canonical differential systems. The general spectral theory is then applied both to classical spectral problems (Sturm-Liouville, Dirac, string equations, Krein systems) and new, important nonlinear equations such as the nonlinear Schrödinger equation, the modified Korteweg-de Vries equation and the sinh-Gordon equation.



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