

Contents

Sections marked with an asterisk are somewhat out of the book's main line of development and may be omitted in a first reading.

| | |
|-----------------------------|------|
| PREFACE TO VOLUME II | xvii |
|-----------------------------|------|

| | |
|-----------------|----|
| NOTATION | xx |
|-----------------|----|

| | |
|--------------------------------------|---|
| 15 NON-ABELIAN GAUGE THEORIES | 1 |
|--------------------------------------|---|

| | |
|------------------------------|---|
| 15.1 Gauge Invariance | 2 |
|------------------------------|---|

Gauge transformations □ Structure constants □ Jacobi identity □ Adjoint representation □ Yang–Mills theory □ Covariant derivatives □ Field strength tensor □ Finite gauge transformations □ Analogy with general relativity

| | |
|--|---|
| 15.2 Gauge Theory Lagrangians and Simple Lie Groups | 7 |
|--|---|

Gauge field Lagrangian □ Metric □ Antisymmetric structure constants □ Simple, semisimple, and $U(1)$ Lie algebras □ Structure of gauge algebra □ Compact algebras □ Coupling constants

| | |
|---|----|
| 15.3 Field Equations and Conservation Laws | 12 |
|---|----|

Conserved currents □ Covariantly conserved currents □ Inhomogeneous field equations □ Homogeneous field equations □ Analogy with energy-momentum tensor □ Symmetry generators

| | |
|--------------------------|----|
| 15.4 Quantization | 14 |
|--------------------------|----|

Primary and secondary first-class constraints □ Axial gauge □ Gribov ambiguity □ Canonical variables □ Hamiltonian □ Reintroduction of A_α^0 □ Covariant action □ Gauge invariance of the measure

| | |
|--|----|
| 15.5 The De Witt–Faddeev–Popov Method | 19 |
|--|----|

Generalization of axial gauge results □ Independence of gauge fixing functionals □ Generalized Feynman gauge □ Form of vertices

| | |
|--------------------|----|
| 15.6 Ghosts | 24 |
|--------------------|----|

Determinant as path integral □ Ghost and antighost fields □ Feynman rules for ghosts □ Modified action □ Power counting and renormalizability

| | |
|---|----|
| 15.7 BRST Symmetry | 27 |
| Auxiliary field h_α □ BRST transformation □ Nilpotence □ Invariance of new action □ BRST-cohomology □ Independence of gauge fixing □ Application to electrodynamics □ BRST-quantization □ Geometric interpretation | |
| 15.8 Generalizations of BRST Symmetry* | 36 |
| De Witt notation □ General Faddeev–Popov–De Witt theorem □ BRST transformations □ New action □ Slavnov operator □ Field-dependent structure constants □ Generalized Jacobi identity □ Invariance of new action □ Independence of gauge fixing □ Beyond quadratic ghost actions □ BRST quantization □ BRST cohomology □ Anti-BRST symmetry | |
| 15.9 The Batalin–Vilkovisky Formalism* | 42 |
| Open gauge algebras □ Antifields □ Master equation □ Minimal fields and trivial pairs □ BRST-transformations with antifields □ Antibrackets □ Anticanonical transformations □ Gauge fixing □ Quantum master equation | |
| Appendix A A Theorem Regarding Lie Algebras | 50 |
| Appendix B The Cartan Catalog | 54 |
| Problems | 58 |
| References | 59 |
| 16 EXTERNAL FIELD METHODS | 63 |
| 16.1 The Quantum Effective Action | 63 |
| Currents □ Generating functional for all graphs □ Generating functional for connected graphs □ Legendre transformation □ Generating functional for one-particle-irreducible graphs □ Quantum-corrected field equations □ Summing tree graphs | |
| 16.2 Calculation of the Effective Potential | 68 |
| Effective potential for constant fields □ One loop calculation □ Divergences □ Renormalization □ Fermion loops | |
| 16.3 Energy Interpretation | 72 |
| Adiabatic perturbation □ Effective potential as minimum energy □ Convexity □ Instability between local minima □ Linear interpolation | |
| 16.4 Symmetries of the Effective Action | 75 |
| Symmetry and renormalization □ Slavnov–Taylor identities □ Linearly realized symmetries □ Fermionic fields and currents | |
| Problems | 78 |
| References | 78 |

| | | |
|-------------|--|------------|
| 17 | RENORMALIZATION OF GAUGE THEORIES | 80 |
| 17.1 | The Zinn-Justin Equation | 80 |
| | Slavnov–Taylor identities for BRST symmetry □ External fields $K_n(x)$ □ Antibrackets | |
| 17.2 | Renormalization: Direct Analysis | 82 |
| | Recursive argument □ BRST-symmetry condition on infinities □ Linearity in $K_n(x)$ □ New BRST symmetry □ Cancellation of infinities □ Renormalization constants □ Nonlinear gauge conditions | |
| 17.3 | Renormalization: General Gauge Theories* | 91 |
| | Are ‘non-renormalizable’ gauge theories renormalizable? □ Structural constraints □ Anticanonical change of variables □ Recursive argument □ Cohomology theorems | |
| 17.4 | Background Field Gauge | 95 |
| | New gauge fixing functions □ True and formal gauge invariance □ Renormalization constants | |
| 17.5 | A One-Loop Calculation in Background Field Gauge | 100 |
| | One-loop effective action □ Determinants □ Algebraic calculation for constant background fields □ Renormalization of gauge fields and couplings □ Interpretation of infinities | |
| | Problems | 109 |
| | References | 110 |
| 18 | RENORMALIZATION GROUP METHODS | 111 |
| 18.1 | Where do the Large Logarithms Come From? | 112 |
| | Singularities at zero mass □ ‘Infrared safe’ amplitudes and rates □ Jets □ Zero mass singularities from renormalization □ Renormalized operators | |
| 18.2 | The Sliding Scale | 119 |
| | Gell-Mann–Low renormalization □ Renormalization group equation □ One-loop calculations □ Application to ϕ^4 theory □ Field renormalization factors □ Application to quantum electrodynamics □ Effective fine structure constant □ Field-dependent renormalized couplings □ Vacuum instability | |
| 18.3 | Varieties of Asymptotic Behavior | 130 |
| | Singularities at finite energy □ Continued growth □ Fixed point at finite coupling □ Asymptotic freedom □ Lattice quantization □ Triviality □ Universal coefficients in the beta function | |

| | |
|--|-----|
| 18.4 Multiple Couplings and Mass Effects | 139 |
| Behavior near a fixed point □ Invariant eigenvalues □ Nonrenormalizable theories □ Finite dimensional critical surfaces □ Mass renormalization at zero mass □ Renormalization group equations for masses | |
| 18.5 Critical Phenomena* | 145 |
| Low wave numbers □ Relevant, irrelevant, and marginal couplings □ Phase transitions and critical surfaces □ Critical temperature □ Behavior of correlation length □ Critical exponent □ $4 - \epsilon$ dimensions □ Wilson–Fisher fixed point □ Comparison with experiment □ Universality classes | |
| 18.6 Minimal Subtraction | 148 |
| Definition of renormalized coupling □ Calculation of beta function □ Applica- tion to electrodynamics □ Modified minimal subtraction □ Non-renormalizable interactions | |
| 18.7 Quantum Chromodynamics | 152 |
| Quark colors and flavors □ Calculation of beta function □ Asymptotic freedom □ Quark and gluon trapping □ Jets □ e^+e^- annihilation into hadrons □ Accidental symmetries □ Non-renormalizable corrections □ Behavior of gauge coupling □ Experimental results for g_s and Λ | |
| 18.8 Improved Perturbation Theory* | 157 |
| Leading logarithms □ Coefficients of logarithms | |
| Problems | 158 |
| References | 159 |
| 19 SPONTANEOUSLY BROKEN GLOBAL SYMMETRIES | 163 |
| 19.1 Degenerate Vacua | 163 |
| Degenerate minima of effective potential □ Broken symmetry or symmetric super- positions? □ Large systems □ Factorization at large distances □ Diagonalization of vacuum expectation values □ Cluster decomposition | |
| 19.2 Goldstone Bosons | 167 |
| Broken global symmetries imply massless bosons □ Proof using effective potential □ Proof using current algebra □ F factors and vacuum expectation values □ Interactions of soft Goldstone bosons | |
| 19.3 Spontaneously Broken Approximate Symmetries | 177 |
| Pseudo-Goldstone bosons □ Tadpoles □ Vacuum alignment □ Mass matrix □ Positivity | |

19.4 Pions as Goldstone Bosons 182

$SU(2) \times SU(2)$ chiral symmetry of quantum chromodynamics □ Breakdown to isospin □ Vector and axial-vector weak currents □ Pion decay amplitude □ Axial form factors of nucleon □ Goldberger-Treiman relation □ Vacuum alignment □ Quark and pion masses □ Soft pion interactions □ Historical note

19.5 Effective Field Theories: Pions and Nucleons 192

Current algebra for two soft pions □ Current algebra justification for effective Lagrangian □ σ -model □ Transformation to derivative coupling □ Nonlinear realization of $SU(2) \times SU(2)$ □ Effective Lagrangian for soft pions □ Direct justification of effective Lagrangian □ General effective Lagrangian for pions □ Power counting □ Pion-pion scattering for massless pions □ Identification of F -factor □ Pion mass terms in effective Lagrangian □ Pion-pion scattering for real pions □ Pion-pion scattering lengths □ Pion-nucleon effective Lagrangian □ Covariant derivatives □ $g_A \neq 1$ □ Power counting with nucleons □ Pion-nucleon scattering lengths □ σ -terms □ Isospin violation □ Adler-Weisberger sum rule

19.6 Effective Field Theories: General Broken Symmetries 211

Transformation to derivative coupling □ Goldstone bosons and right cosets □ Symmetric spaces □ Cartan decomposition □ Nonlinear transformation rules □ Uniqueness □ Covariant derivatives □ Symmetry breaking terms □ Application to quark mass terms □ Power counting □ Order parameters

19.7 Effective Field Theories: $SU(3) \times SU(3)$ 225

$SU(3)$ multiplets and matrices □ Goldstone bosons of broken $SU(3) \times SU(3)$ □ Quark mass terms □ Pseudoscalar meson masses □ Electromagnetic corrections □ Quark mass ratios □ Higher terms in Lagrangian □ Nucleon mass shifts

19.8 Anomalous Terms in Effective Field Theories* 234

Wess-Zumino-Witten term □ Five-dimensional form □ Integer coupling □ Uniqueness and de Rham cohomology

19.9 Unbroken Symmetries 238

Persistent mass conjecture □ Vafa-Witten proof □ Small non-degenerate quark masses

19.10 The $U(1)$ Problem 243

Chiral $U(1)$ symmetry □ Implications for pseudoscalar masses

Problems 246

References 247

| | | |
|-----------------|---|------------|
| 20 | OPERATOR PRODUCT EXPANSIONS | 252 |
| 20.1 | The Expansion: Description and Derivation | 253 |
| | Statement of expansion □ Dominance of simple operators □ Path-integral derivation | |
| 20.2 | Momentum Flow* | 255 |
| | ϕ^2 contribution for two large momenta □ Renormalized operators □ Integral equation for coefficient function □ ϕ^2 contribution for many large momenta | |
| 20.3 | Renormalization Group Equations for Coefficient Functions | 263 |
| | Derivation and solution □ Behavior for fixed points □ Behavior for asymptotic freedom | |
| 20.4 | Symmetry Properties of Coefficient Functions | 265 |
| | Invariance under spontaneously broken symmetries | |
| 20.5 | Spectral Function Sum Rules | 266 |
| | Spectral functions defined □ First, second, and third sum rules □ Application to chiral $SU(N) \times SU(N)$ □ Comparison with experiment | |
| 20.6 | Deep Inelastic Scattering | 272 |
| | Form factors W_1 and W_2 □ Deep inelastic differential cross section □ Bjorken scaling □ Parton model □ Callan–Gross relation □ Sum rules □ Form factors T_1 and T_2 □ Relation between T_r and W_r □ Symmetric tensor operators □ Twist □ Operators of minimum twist □ Calculation of coefficient functions □ Sum rules for parton distribution functions □ Altarelli–Parisi differential equations □ Logarithmic corrections to Bjorken scaling | |
| 20.7 | Renormalons* | 283 |
| | Borel summation of perturbation theory □ Instanton and renormalon obstructions □ Instantons in massless ϕ^4 theory □ Renormalons in quantum chromodynamics | |
| Appendix | Momentum Flow: The General Case | 288 |
| | Problems | 292 |
| | References | 293 |
| 21 | SPONTANEOUSLY BROKEN GAUGE SYMMETRIES | 295 |
| 21.1 | Unitarity Gauge | 295 |
| | Elimination of Goldstone bosons □ Vector boson masses □ Unbroken symmetries and massless vector bosons □ Complex representations □ Vector field propagator □ Continuity for vanishing gauge couplings | |

| | |
|--|-----|
| 21.2 Renormalizable ξ-Gauges | 300 |
| Gauge fixing function \square Gauge-fixed Lagrangian \square Propagators | |
| 21.3 The Electroweak Theory | 305 |
| Lepton-number preserving symmetries $\square SU(2) \times U(1) \square W^\pm, Z^0$, and photons \square Mixing angle \square Lepton-vector boson couplings $\square W^\pm$ and Z^0 masses \square Muon decay \square Effective fine structure constant \square Discovery of neutral currents \square Quark currents \square Cabibbo angle $\square c$ quark \square Third generation \square Kobayashi–Maskawa matrix \square Discovery of W^\pm and $Z^0 \square$ Precise experimental tests \square Accidental symmetries \square Nonrenormalizable corrections \square Lepton nonconservation and neutrino masses \square Baryon nonconservation and proton decay | |
| 21.4 Dynamically Broken Local Symmetries* | 318 |
| Fictitious gauge fields \square Construction of Lagrangian \square Power counting \square General mass formula \square Example: $SU(2) \times SU(2) \square$ Custodial $SU(2) \times SU(2) \square$ Technicolor | |
| 21.5 Electroweak–Strong Unification | 327 |
| Simple gauge groups \square Relations among gauge couplings \square Renormalization group flow \square Mixing angle and unification mass \square Baryon and lepton nonconservation | |
| 21.6 Superconductivity* | 332 |
| $U(1)$ broken to $Z_2 \square$ Goldstone mode \square Effective Lagrangian \square Conservation of charge \square Meissner effect \square Penetration depth \square Critical field \square Flux quantization \square Zero resistance \square ac Josephson effect \square Landau–Ginzburg theory \square Correlation length \square Vortex lines $\square U(1)$ restoration \square Stability \square Type I and II superconductors \square Critical fields for vortices \square Behavior near vortex center \square Effective theory for electrons near Fermi surface \square Power counting \square Introduction of pair field \square Effective action \square Gap equation \square Renormalization group equations \square Conditions for superconductivity | |
| Appendix General Unitarity Gauge | 352 |
| Problems | 353 |
| References | 354 |
| 22 ANOMALIES | 359 |
| 22.1 The π^0 Decay Problem | 359 |
| Rate for $\pi^0 \rightarrow 2\gamma \square$ Naive estimate \square Suppression by chiral symmetry \square Comparison with experiment | |
| 22.2 Transformation of the Measure: The Abelian Anomaly | 362 |
| Chiral and non-chiral transformations \square Anomaly function \square Chern–Pontryagin density \square Nonconservation of current \square Conservation of gauge-non-invariant | |

current \square Calculation of $\pi^0 \rightarrow 2\gamma$ \square Euclidean calculation \square Atiyah–Singer index theorem

22.3 Direct Calculation of Anomalies: The General Case 370

Fermion non-conserving currents \square Triangle graph calculation \square Shift vectors \square Symmetric anomaly \square Bardeen form \square Adler–Bardeen theorem \square Massive fermions \square Another approach \square Global anomalies

22.4 Anomaly-Free Gauge Theories 383

Gauge anomalies must vanish \square Real and pseudoreal representations \square Safe groups \square Anomaly cancellation in standard model \square Gravitational anomalies \square Hypercharge assignments \square Another $U(1)$?

22.5 Massless Bound States* 389

Composite quarks and leptons? \square Unbroken chiral symmetries \square 't Hooft anomaly matching conditions \square Anomaly matching for unbroken chiral $SU(n) \times SU(n)$ with $SU(N)$ gauge group \square The case $N = 3$ \square Chiral $SU(3) \times SU(3)$ must be broken \square 't Hooft decoupling condition \square Persistent mass condition

22.6 Consistency Conditions 396

Wess–Zumino conditions \square BRST cohomology \square Derivation of symmetric anomaly \square Descent equations \square Solution of equations \square Schwinger terms \square Anomalies in Zinn–Justin equation \square Antibracket cohomology \square Algebraic proof of anomaly absence for safe groups

22.7 Anomalies and Goldstone Bosons 408

Anomaly matching \square Solution of anomalous Slavnov–Taylor identities \square Uniqueness \square Anomalous Goldstone boson interactions \square The case $SU(3) \times SU(3)$ \square Derivation of Wess–Zumino–Witten interaction \square Evaluation of integer coefficient \square Generalization

Problems 416

References 417

23 EXTENDED FIELD CONFIGURATIONS 421

23.1 The Uses of Topology 422

Topological classifications \square Homotopy \square Skyrmions \square Derrick's theorem \square Domain boundaries \square Bogomol'nyi inequality \square Cosmological problems \square Instantons \square Monopoles and vortex lines \square Symmetry restoration

23.2 Homotopy Groups 430

Multiplication rule for $\pi_1(\mathcal{M})$ \square Associativity \square Inverses \square $\pi_1(S_1)$ \square Topological conservation laws \square Multiplication rule for $\pi_k(\mathcal{M})$ \square Winding number

| | |
|---|-----|
| 23.3 Monopoles | 436 |
| <i>SU(2)/U(1)</i> model □ Winding number □ Electromagnetic field □ Magnetic monopole moment □ Kronecker index □ 't Hooft–Polyakov monopole □ Another Bogomol'nyi inequality □ BPS monopole □ Dirac gauge □ Charge quantization □ $G/(H' \times U(1))$ monopoles □ Cosmological problems □ Monopole–particle interactions □ G/H monopoles with G not simply connected □ Irrelevance of field content | |
| 23.4 The Cartan–Maurer Integral Invariant | 445 |
| Definition of the invariant □ Independence of coordinate system □ Topological invariance □ Additivity □ Integral invariant for $S_1 \mapsto U(1)$ □ Bott's theorem □ Integral invariant for $S_3 \mapsto SU(2)$ | |
| 23.5 Instantons | 450 |
| Evaluation of Cartan–Maurer invariant □ Chern–Pontryagin density □ One more Bogomol'nyi inequality □ $\nu = 1$ solution □ General winding number □ Solution of $U(1)$ problem □ Baryon and lepton non-conservation by electroweak instantons □ Minkowskian approach □ Barrier penetration □ Thermal fluctuations | |
| 23.6 The Theta Angle | 455 |
| Cluster decomposition □ Superposition of winding numbers □ P and CP non-conservation □ Complex fermion masses □ Suppression of P and CP non-conservation by small quark masses □ Neutron electric dipole moment □ Peccei–Quinn symmetry □ Axions □ Axion mass □ Axion interactions | |
| 23.7 Quantum Fluctuations around Extended Field Configurations | 462 |
| Fluctuations in general □ Collective parameters □ Determinantal factor □ Coupling constant dependence □ Counting collective parameters | |
| 23.8 Vacuum Decay | 464 |
| False and true vacua □ Bounce solutions □ Four dimensional rotational invariance □ Sign of action □ Decay rate per volume □ Thin wall approximation | |
| Appendix A Euclidean Path Integrals | 468 |
| Appendix B A List of Homotopy Groups | 472 |
| Problems | 473 |
| References | 474 |
| AUTHOR INDEX | 478 |
| SUBJECT INDEX | 484 |