

Contents

	<i>Preface to the 25th Anniversary Edition</i>	xi
8	One-loop diagrams in the bosonic string theory	1
8.1	Open-string one-loop amplitudes	3
8.1.1	The planar diagrams	7
8.1.2	The nonorientable diagrams	27
8.1.3	Nonplanar loop diagrams	32
8.2	Closed-string one-loop amplitudes	37
8.2.1	The torus	38
8.2.2	Modular invariance	44
8.2.3	The integration region	49
8.2.4	Analysis of divergences	51
8.2.5	The cosmological constant	54
8.2.6	Amplitudes with closed-string massless states	57
8.3	Other diagrams for unoriented strings	58
8.3.1	Higher-order tree diagrams	58
8.3.2	The real projective plane	63
8.3.3	Other loop diagrams	66
8.4	Summary	68
8.A	Jacobi Θ functions	69
9	One-loop diagrams in superstring theory	75
9.1	Open-superstring amplitudes	76
9.1.1	Amplitudes with $M < 4$ massless external states	78
9.1.2	The planar diagrams	79
9.1.3	Nonorientable diagrams	85
9.1.4	Orientable nonplanar diagrams	87
9.2	Type II theories	89
9.2.1	Finiteness of the torus amplitude	90
9.2.2	Compactification on a torus	94
9.2.3	The low-energy limit of one-loop amplitudes	98
9.3	The heterotic string theory	102
9.3.1	The torus with four external particles	103
9.3.2	Modular invariance of the $E_8 \times E_8$ and $SO(32)$ theories	108

9.4	Calculations in the RNS formalism	112
9.4.1	Modular invariance and the GSO projection	112
9.4.2	The loop calculations	117
9.5	Orbifolds and twisted strings	119
9.5.1	Generalization of the GSO projection	119
9.5.2	Strings on orbifolds	121
9.5.3	Twisted strings in ten dimensions	126
9.5.4	Alternative view of the $SO(16) \times SO(16)$ theory	130
9.6	Summary	133
9.A	Traces of fermionic zero modes	134
9.B	Modular invariance of the functions F_2 and $\mathcal{L}$	137
10	The gauge anomaly in type I superstring theory	141
10.1	Introduction to anomalies	143
10.1.1	Anomalies in point-particle field theory	143
10.1.2	The gauge anomaly in $D = 10$ super Yang-Mills theory	146
10.1.3	Anomalies in superstring theory	149
10.2	Analysis of hexagon diagrams	152
10.2.1	The planar diagram anomaly	154
10.2.2	The anomaly in the nonorientable diagram	164
10.2.3	Absence of anomalies in nonplanar diagrams	165
10.3	Other one-loop anomalies in superstring theory	167
10.4	Cancellation of divergences for $SO(32)$	168
10.4.1	Dilaton tadpoles and loop divergences	169
10.4.2	Divergence cancellations	171
10.5	Summary	172
10.A	An alternative regulator	173
11	Functional methods in the light-cone gauge	179
11.1	The string path integral	179
11.1.1	The analog model	180
11.1.2	The free string propagator	181
11.1.3	A lattice cutoff	184
11.1.4	The continuum limit	189
11.2	Amplitude calculations	191
11.2.1	Interaction vertices	191
11.2.2	Parametrization of scattering processes	195
11.2.3	Evaluation of the functional integral	197
11.2.4	Amplitudes with external ground states	201
11.3	Open-string tree amplitudes	203
11.3.1	The conformal mapping	204
11.3.2	Evaluation of amplitudes	209

11.4	Open-string trees with excited external states	212
11.4.1	The Green function on an infinite strip	213
11.4.2	Green functions for arbitrary tree amplitudes	214
11.4.3	The amplitude in terms of oscillators	216
11.4.4	The general form of the Neumann coefficients	218
11.4.5	The Neumann coefficients for the cubic open-string vertex	219
11.5	One-loop open-string amplitudes	223
11.5.1	The conformal mapping for the planar loop diagram	223
11.5.2	The Green function	225
11.5.3	The planar one-loop amplitude	226
11.5.4	Other one-loop amplitudes	227
11.6	Closed-string amplitudes	229
11.6.1	Tree amplitudes	229
11.6.2	Closed-string one-loop amplitudes	231
11.7	Superstrings	235
11.7.1	The $SU(4) \times U(1)$ formalism	235
11.7.2	The super-Poincaré generators	239
11.7.3	Supersymmetry algebra in the interacting theory	244
11.7.4	The continuity delta functional	246
11.7.5	Singular operators near the interaction point	248
11.7.6	The interaction terms	251
11.7.7	Tree amplitudes for open superstrings	254
11.8	Summary	255
11.A	The determinant of the Laplacian	256
11.B	The Jacobian for the conformal transformation	265
11.C	Properties of the functions f_m	267
11.D	Properties of the $SU(4)$ Clebsch–Gordan coefficients	269
12	Some differential geometry	271
12.1	Spinors in general relativity	271
12.2	Spin structures on the string world sheet	277
12.3	Topologically nontrivial gauge fields	281
12.3.1	The tangent bundle	281
12.3.2	Gauge fields and vector bundles	283
12.4	Differential forms	287
12.5	Characteristic classes	298
12.5.1	The nonabelian case	300
12.5.2	Characteristic classes of manifolds	302
12.5.3	The Euler characteristic of a Riemann surface	304
13	Low-energy effective action	307
13.1	Minimal supergravity plus super Yang–Mills	308

13.1.1	$N = 1$ supergravity in ten and eleven dimensions	310
13.1.2	Type IIB supergravity	314
13.1.3	The coupled supergravity super Yang–Mills system	321
13.2	Scale invariance of the classical theory	326
13.3	Anomaly analysis	330
13.3.1	Structure of field theory anomalies	333
13.3.2	Gravitational anomalies	336
13.3.3	Mixed anomalies	338
13.3.4	The anomalous Feynman diagrams	340
13.3.5	Mathematical characterization of anomalies	343
13.3.6	Other types of anomalies	346
13.4	Explicit formulas for the anomalies	347
13.5	Anomaly cancellations	351
13.5.1	Type I supergravity without matter	352
13.5.2	Type IIB supergravity	352
13.5.3	Allowed gauge groups for $N = 1$ superstring theories	353
13.5.4	The $SO(16) \times SO(16)$ theory	358
14	Compactification of higher dimensions	363
14.1	Wave operators in ten dimensions	363
14.1.1	Massless fields in ten dimensions	363
14.1.2	Zero modes of wave operators	365
14.2	Massless fermions	367
14.2.1	The index of the Dirac operator	369
14.2.2	Incorporation of gauge fields	371
14.2.3	The chiral asymmetry	374
14.2.4	The Rarita–Schwinger operator	375
14.2.5	Outlook	375
14.3	Zero modes of antisymmetric tensor fields	376
14.3.1	Antisymmetric tensor fields	377
14.3.2	Application to axions in $N = 1$ superstring theory	380
14.3.3	The ‘nonzero modes’	386
14.3.4	The exterior derivative and the Dirac operator	387
14.4	Index theorems on the string world sheet	392
14.4.1	The Dirac index	392
14.4.2	The Euler characteristic	394
14.4.3	Zero modes of conformal ghosts	396
14.4.4	Zero modes of superconformal ghosts	397
14.5	Zero modes of nonlinear fields	397
14.6	Models of the fermion quantum numbers	404
14.7	Anomaly cancellation in four dimensions	410
15	Some algebraic geometry	413

15.1	Low-energy supersymmetry	413
15.1.1	Motivation	413
15.1.2	Conditions for unbroken supersymmetry	415
15.1.3	Manifolds of $SU(3)$ holonomy	418
15.2	Complex manifolds	420
15.2.1	Almost complex structure	421
15.2.2	The Nijenhuis tensor	421
15.2.3	Examples of complex manifolds	424
15.3	Kähler manifolds	430
15.3.1	The Kähler metric	430
15.3.2	Exterior derivatives	431
15.3.3	The affine connection and the Riemann tensor	432
15.3.4	Examples of Kähler manifolds	434
15.4	Ricci-flat Kähler manifolds and $SU(N)$ holonomy	437
15.4.1	The Calabi–Yau metric	438
15.4.2	Covariantly constant forms	440
15.4.3	Some manifolds of $SU(N)$ holonomy	441
15.5	Wave operators on Kähler manifolds	444
15.5.1	The Dirac operator	444
15.5.2	Dolbeault cohomology	447
15.5.3	The Hodge decomposition	449
15.5.4	Hodge numbers	451
15.6	Yang–Mills equations and holomorphic vector bundles	453
15.6.1	Holomorphic vector bundles	454
15.6.2	The Donaldson–Uhlenbeck–Yau equation	456
15.6.3	Examples	458
15.7	Dolbeault cohomology and some applications	463
15.7.1	Zero modes of the Dirac operator	465
15.7.2	Deformations of complex manifolds	467
15.7.3	Deformations of holomorphic vector bundles	469
15.8	Branched coverings of complex manifolds	471
16	Models of low-energy supersymmetry	475
16.1	A simple Ansatz	475
16.2	The spectrum of massless particles	480
16.2.1	Zero modes of charged fields	481
16.2.2	Fluctuations of the gravitational field	483
16.2.3	The other Bose fields	486
16.3	Symmetry breaking by Wilson lines	486
16.3.1	Symmetry breaking patterns	488
16.3.2	A four generation model	491
16.4	Relation to conventional grand unification	495

16.4.1	Alternative description of symmetry breaking	496
16.4.2	E_6 relations among coupling constants	497
16.4.3	Counting massless particles	500
16.4.4	Fractional electric charges	506
16.4.5	Discussion	510
16.5	Global symmetries	511
16.5.1	CP conservation in superstring models	511
16.5.2	R transformations in superstring models	512
16.5.3	Global symmetries of the toy model	513
16.5.4	Transformation laws of matter fields	518
16.6	Topological formulas for Yukawa couplings	520
16.6.1	A topological formula for the superpotential	522
16.6.2	The kinetic terms	527
16.6.3	A nonrenormalization theorem and its consequences	528
16.6.4	Application to the toy model	532
16.7	Another approach to symmetry breaking	534
16.8	Discussion	541
16.9	Renormalization of coupling constants	544
16.10	Orbifolds and algebraic geometry	548
16.11	Outlook	551
	<i>Bibliography</i>	553
	<i>Index</i>	591