

CONTENTS

PREFACE

xiii

I. INTRODUCTION

1

- 1.1 Particles and Interactions
- 1.2 Gauge Theories of Interactions
- 1.3 Notations and Conventions

1

6

10

II. QUARKS

12

2.1 Internal Symmetries

12

- 1 Isospin
- 2 The gauge groups
- 3 More general internal symmetries: $SU(n)$
- 4 Unitary symmetry

13

14

14

15

2.2 Representation of $SU(3)$

17

- 1 The basic representation
- 2 Young's tableaux
- 3 Irreducible representations

17

18

20

2.3 The Quark Model

22

- 1 Quarks as basic triplets
- 2 Quarks as building blocks
- 3 Weight diagrams
- 4 The composition of hadrons

22

25

25

28

2.4 Color

28

- 1 Independent quark model
- 2 Color $SU(3)$ group

28

30

2.5 Electromagnetic and Weak Probes

33

- 1 Electromagnetic interactions
- 2 Parton model
- 3 Evidence for color
- 4 Weak interactions

33

35

38

40

2.6 Charm

43

- 1 The charmed quark
- 2 The J/ψ and its family
- 3 Correspondence between quarks and leptons

43

45

46

III. MAXWELL FIELD: $U(1)$ GAUGE THEORY

47

- 3.1 Global and Local Gauge Invariance
- 3.2 Spontaneous Breaking of Global Gauge Invariance: Goldstone Mode
- 3.3 Spontaneous Breaking of Local Gauge Invariance: Higgs Mode
- 3.4 Classical Finite-Energy Solutions

47

50

53

55

3.5	Magnetic Flux Quantization	56
3.6	Soliton Solutions: Vortex Lines	58
IV.	YANG-MILLS FIELDS: NON-ABELIAN GAUGE THEORIES	61
4.1	Introductory Note	61
4.2	Lie Groups	61
1	Structure constants	61
2	Matrix representations	62
3	Topological properties	64
4	General remarks	66
4.3	The Yang-Mills Constructions	67
1	Global gauge invariance	67
2	Local gauge invariance	69
4.4	Properties of Yang-Mills Fields	72
1	Electric and magnetic fields	72
2	Dual tensor	73
3	Path representation of the gauge group	74
4.5	Canonical Formalism	77
1	Equations of motion	77
2	Hamiltonian	79
4.6	Spontaneous Symmetry Breaking	80
1	The little group	80
2	Higgs mechanism	83
V.	TOPOLOGICAL SOLITONS	86
5.1	Solitons	86
5.2	The Instanton	88
1	Topological charge	88
2	Explicit solution	92
5.3	The Monopole	94
1	Topological stability	94
2	Flux quantization	95
3	Boundary conditions	98
4	Explicit solution	100
5	Physical fields	101
6	Spin from isospin	103
VI.	WEINBERG-SALAM MODEL	105
6.1	The Matter Fields	105
6.2	The Gauge Fields	108
1	Gauging $SU(2) \times U(1)$	108
2	Determination of constants	111
3	Interactions	112
6.3	The General Theory	113
1	Mass terms	113
2	Cabibbo angle	117
3	Kobayashi-Maskawa matrix	118
4	Solitons	119

VII. METHOD OF PATH INTEGRALS	121
7.1 Non-Relativistic Quantum Mechanics	121
7.2 Quantum Field Theory	126
7.3 External Sources	128
7.4 Euclidean 4-Space	132
7.5 Calculation of Path Integrals	134
7.6 The Feynman Propagator	135
7.7 Feynman Graphs	137
7.8 Boson Loops and Fermion Loops	140
7.9 Fermion Fields	142
VIII. QUANTIZATION OF GAUGE FIELDS	147
8.1 Canonical Quantization	147
1 Free Maxwell field	147
2 Pure Yang-Mills fields	150
8.2 Path Integral Method in Hamiltonian Form	150
8.3 Feynman Path Integral: Fadeev-Popov Method	152
8.4 Free Maxwell Field	156
1 Lorentz gauge	156
2 Coulomb gauge	159
3 Temporal and axial gauges	160
8.5 Pure Yang-Mills Fields	162
1 Axial gauge	163
2 Lorentz gauge: Fadeev-Popov ghosts	164
8.6 The θ -World and the Instanton	165
1 Discovering the θ -world	165
2 Instanton as tunneling solution	168
3 The θ -action	171
8.7 Gribov Ambiguity	172
8.8 Projection Operator for Gauss' Law	174
IX. RENORMALIZATION	177
9.1 Charge Renormalization	177
9.2 Perturbative Renormalization in Quantum Electrodynamics	180
9.3 The Renormalization Group	183
1 Scale transformations	183
2 Scaling form	184
3 Fixed points	185
4 Callan-Symanzik equation	186
9.4 Scalar Fields	188
1 Renormalizability	188
2 ϕ^4 theory	189
3 "Triviality" and the Landau ghost	191
9.5 The Physics of Renormalization	192
1 Renormalization-group transformation	192
2 Real-space renormalization	195
3 Fixed points and relevancy	197
4 Renormalization and universality	199

Appendix to Chapter 9. Renormalization of QED	201
1 Vertex	201
2 Electron Propagator	201
3 Photon Propagator	202
4 Scaling Properties	205
5 Renormalization	206
6 Gauge Invariance and the Photon Mass	208
X. METHOD OF EFFECTIVE POTENTIAL	210
10.1 Spontaneous Symmetry Breaking	210
10.2 The Effective Action	210
10.3 The Effective Potential	212
10.4 The Loop Expansion	215
10.5 One-Loop Effective Potential	218
10.6 Renormalization	219
1 General scheme	219
2 Massive case	221
3 Massless case	221
10.7 Dimensional Transmutation	223
10.8 A Non-Relativistic Example	225
10.9 Application to Weinberg-Salam Model	227
XI. THE AXIAL ANOMALY	230
11.1 Origin of the Axial Anomaly	230
11.2 The Triangle Graph	231
11.3 Anomalous Divergence of the Chiral Current	237
11.4 Physical Explanation of the Axial Anomaly	238
11.5 Cancellation of Anomalies	242
11.6 't Hooft's Principle	247
XII. QUANTUM CHROMODYNAMICS	252
12.1 General Properties	252
1 Lagrangian density	252
2 Feynman rules	253
3 Quark-gluon interactions	255
4 Gluon self-interactions	256
12.2 The Color Gyromagnetic Ratio	260
12.3 Asymptotic Freedom	262
1 The running coupling constant	262
2 The vacuum as magnetic medium	265
3 The Nielsen-Hughes formula	268
12.4 The Pion as Goldstone Boson	269
1 The low-energy domain	269
2 Chiral symmetry: an idealized limit	269
3 PCAC	272
4 The decay $\pi^0 \rightarrow 2\gamma$	273
5 Extension to pion octet	275
12.5 The $U(1)$ Puzzle	276

PREFACE

12.6 θ -Worlds in QCD	278
1 Euclidean action	278
2 The axial anomaly and the index theorem	279
3 Chiral limit: Collapse of the θ -worlds	282
4 Quark mass matrix	283
5 Strong CP violation	286
XIII. LATTICE GAUGE THEORY	288
13.1 Wilson's Lattice Action	288
13.2 Transfer Matrix	291
13.3 Lattice Hamiltonian	293
13.4 Lattice Fermions	297
13.5 Wilson Loop and Confinement	299
13.6 Continuum Limit	303
13.7 Monte Carlo Methods	304
XIV. QUARK CONFINEMENT	308
14.1 Wilson Criterion and Electric Confinement	308
14.2 String Model of Hadrons	311
14.3 Superconductivity: Magnetic Confinement	312
1 Experimental manifestation	312
2 Theory	313
3 Mechanism for monopole confinement	314
14.4 Electric and Magnetic Order Parameters	316
14.5 Scenario for Quark Confinement	319
Appendix to Chapter 14. Symmetry and Confinement	322
1 Quark Propagator	322
2 Center Symmetry	324
3 Confinement as Symmetry	325
INDEX	327

I owe special thanks to my colleagues at M.I.T., especially A. Guth, R. Johnson, K. Johnson, and J. Polonyi, from whom I have learned much that is being passed along in this book.

Kerson Huang

Whitehead, Mass.
February 1991