

# Contents

	Preface . . . . .	xi
	Notations and Conventions . . . . .	xix
	Editor's Foreword . . . . .	xxii
<b>Part I: Feynman Diagrams and Quantum Electrodynamics</b>		
<b>1</b>	<b>Invitation: Pair Production in <math>e^+e^-</math> Annihilation</b> . . . . .	<b>3</b>
<b>2</b>	<b>The Klein-Gordon Field</b> . . . . .	<b>13</b>
	2.1 The Necessity of the Field Viewpoint . . . . .	13
	2.2 Elements of Classical Field Theory . . . . .	15
	<i>Lagrangian Field Theory; Hamiltonian Field Theory;</i> <i>Noether's Theorem</i>	
	2.3 The Klein-Gordon Field as Harmonic Oscillators . . . . .	19
	2.4 The Klein-Gordon Field in Space-Time . . . . .	25
	<i>Causality; The Klein-Gordon Propagator;</i> <i>Particle Creation by a Classical Source</i>	
	Problems . . . . .	33
<b>3</b>	<b>The Dirac Field</b> . . . . .	<b>35</b>
	3.1 Lorentz Invariance in Wave Equations . . . . .	35
	3.2 The Dirac Equation . . . . .	40
	<i>Weyl Spinors</i>	
	3.3 Free-Particle Solutions of the Dirac Equation . . . . .	45
	<i>Spin Sums</i>	
	3.4 Dirac Matrices and Dirac Field Bilinears . . . . .	49
	3.5 Quantization of the Dirac Field . . . . .	52
	<i>Spin and Statistics; The Dirac Propagator</i>	
	3.6 Discrete Symmetries of the Dirac Theory . . . . .	64
	<i>Parity; Time Reversal; Charge Conjugation</i>	
	Problems . . . . .	71

<b>4</b>	<b>Interacting Fields and Feynman Diagrams</b> . . . . .	<b>77</b>
4.1	Perturbation Theory—Philosophy and Examples . . . . .	77
4.2	Perturbation Expansion of Correlation Functions . . . . .	82
4.3	Wick's Theorem . . . . .	88
4.4	Feynman Diagrams . . . . .	90
4.5	Cross Sections and the $S$ -Matrix . . . . .	99
4.6	Computing $S$ -Matrix Elements from Feynman Diagrams . . . . .	108
4.7	Feynman Rules for Fermions . . . . .	115
	<i>Yukawa Theory</i>	
4.8	Feynman Rules for Quantum Electrodynamics . . . . .	123
	<i>The Coulomb Potential</i>	
	Problems . . . . .	126
<b>5</b>	<b>Elementary Processes of Quantum Electrodynamics</b> . . . . .	<b>131</b>
5.1	$e^+e^- \rightarrow \mu^+\mu^-$ : Introduction . . . . .	131
	<i>Trace Technology; Unpolarized Cross Section; <math>e^+e^- \rightarrow</math> Hadrons</i>	
5.2	$e^+e^- \rightarrow \mu^+\mu^-$ : Helicity Structure . . . . .	141
5.3	$e^+e^- \rightarrow \mu^+\mu^-$ : Nonrelativistic Limit . . . . .	146
	<i>Bound States; Vector Meson Production and Decay</i>	
5.4	Crossing Symmetry . . . . .	153
	<i>Electron-Muon Scattering; Mandelstam Variables</i>	
5.5	Compton Scattering . . . . .	158
	<i>Photon Polarization Sums; The Klein-Nishina Formula;</i>	
	<i>High-Energy Behavior; Pair Annihilation into Photons</i>	
	Problems . . . . .	169
<b>6</b>	<b>Radiative Corrections: Introduction</b> . . . . .	<b>175</b>
6.1	Soft Bremsstrahlung . . . . .	176
	<i>Classical Computation; Quantum Computation</i>	
6.2	The Electron Vertex Function: Formal Structure . . . . .	184
6.3	The Electron Vertex Function: Evaluation . . . . .	189
	<i>Feynman Parameters; Precision Tests of QED</i>	
6.4	The Electron Vertex Function: Infrared Divergence . . . . .	199
*6.5	Summation and Interpretation of Infrared Divergences . . . . .	202
	Problems . . . . .	208
<b>7</b>	<b>Radiative Corrections: Some Formal Developments</b> . . . . .	<b>211</b>
7.1	Field-Strength Renormalization . . . . .	211
	<i>The Electron Self-Energy</i>	
7.2	The LSZ Reduction Formula . . . . .	222
7.3	The Optical Theorem . . . . .	230
	<i>The Optical Theorem for Feynman Diagrams; Unstable Particles</i>	
7.4	The Ward-Takahashi Identity . . . . .	238
7.5	Renormalization of the Electric Charge . . . . .	244
	<i>Dimensional Regularization</i>	
	Problems . . . . .	257
	<b>Final Project: Radiation of Gluon Jets</b> . . . . .	<b>259</b>

**Part II: Renormalization**

<b>8</b>	<b>Invitation: Ultraviolet Cutoffs and Critical Fluctuations</b>	<b>265</b>
<b>9</b>	<b>Functional Methods</b>	<b>275</b>
9.1	Path Integrals in Quantum Mechanics	275
9.2	Functional Quantization of Scalar Fields <i>Correlation Functions; Feynman Rules; Functional Derivatives and the Generating Functional</i>	282
9.3	Quantum Field Theory and Statistical Mechanics	292
9.4	Quantization of the Electromagnetic Field	294
9.5	Functional Quantization of Spinor Fields <i>Anticommuting Numbers; The Dirac Propagator; Generating Functional for the Dirac Field; QED; Functional Determinants</i>	298
*9.6	Symmetries in the Functional Formalism <i>Equations of Motion; Conservation Laws; The Ward- Takahashi Identity</i>	306
	Problems	312
<b>10</b>	<b>Systematics of Renormalization</b>	<b>315</b>
10.1	Counting of Ultraviolet Divergences	315
10.2	Renormalized Perturbation Theory <i>One-Loop Structure of <math>\phi^4</math> Theory</i>	323
10.3	Renormalization of Quantum Electrodynamics <i>One-Loop Structure of QED</i>	330
10.4	Renormalization Beyond the Leading Order	335
*10.5	A Two-Loop Example	338
	Problems	344
<b>11</b>	<b>Renormalization and Symmetry</b>	<b>347</b>
11.1	Spontaneous Symmetry Breaking <i>The Linear Sigma Model; Goldstone's Theorem</i>	348
*11.2	Renormalization and Symmetry: An Explicit Example	352
*11.3	The Effective Action	364
*11.4	Computation of the Effective Action <i>The Effective Action in the Linear Sigma Model</i>	370
*11.5	The Effective Action as a Generating Functional	379
*11.6	Renormalization and Symmetry: General Analysis <i>Goldstone's Theorem Revisited</i>	383
	Problems	389
<b>12</b>	<b>The Renormalization Group</b>	<b>393</b>
12.1	Wilson's Approach to Renormalization Theory	394
12.2	The Callan-Symanzik Equation <i>Renormalization Conditions; The Callan-Symanzik Equation; Computation of <math>\beta</math> and <math>\gamma</math>; The Meaning of <math>\beta</math> and <math>\gamma</math></i>	406

12.3	Evolution of Coupling Constants . . . . .	418
	<i>Solution of the Callan-Symanzik Equation; An Application to QED; Alternatives for the Running of Coupling Constants</i>	
*12.4	Renormalization of Local Operators . . . . .	428
*12.5	Evolution of Mass Parameters . . . . .	432
	<i>Critical Exponents: A First Look</i>	
	Problems . . . . .	438
<b>13</b>	<b>Critical Exponents and Scalar Field Theory . . . . .</b>	<b>439</b>
*13.1	Theory of Critical Exponents . . . . .	440
	<i>Exponents of the Spin Correlation Function; Exponents of Thermodynamic Functions; Values of the Critical Exponents</i>	
*13.2	Critical Behavior in Four Dimensions . . . . .	451
*13.3	The Nonlinear Sigma Model . . . . .	454
	Problems . . . . .	466
	<b>*Final Project: The Coleman-Weinberg Potential . . . . .</b>	<b>469</b>
 <b>Part III: Non-Abelian Gauge Theories</b> 		
<b>14</b>	<b>Invitation: The Parton Model of Hadron Structure . . . . .</b>	<b>473</b>
<b>15</b>	<b>Non-Abelian Gauge Invariance . . . . .</b>	<b>481</b>
15.1	The Geometry of Gauge Invariance . . . . .	482
15.2	The Yang-Mills Lagrangian . . . . .	486
*15.3	The Gauge-Invariant Wilson Loop . . . . .	491
15.4	Basic Facts About Lie Algebras . . . . .	495
	<i>Classification of Lie Algebras; Representations; The Casimir Operator</i>	
	Problems . . . . .	502
<b>16</b>	<b>Quantization of Non-Abelian Gauge Theories . . . . .</b>	<b>505</b>
16.1	Interactions of Non-Abelian Gauge Bosons . . . . .	506
	<i>Feynman Rules for Fermions and Gauge Bosons; Equality of Coupling Constants; A Flaw in the Argument</i>	
16.2	The Faddeev-Popov Lagrangian . . . . .	512
16.3	Ghosts and Unitarity . . . . .	515
*16.4	BRST Symmetry . . . . .	517
*16.5	One-Loop Divergences of Non-Abelian Gauge Theory . . . . .	521
	<i>The Gauge Boson Self-Energy; The <math>\beta</math> Function; Relations among Counterterms</i>	
*16.6	Asymptotic Freedom: The Background Field Method . . . . .	533
16.7	Asymptotic Freedom: A Qualitative Explanation . . . . .	541
	Problems . . . . .	544

<b>17</b>	<b>Quantum Chromodynamics</b>	<b>545</b>
17.1	From Quarks to QCD	545
17.2	$e^+e^-$ Annihilation into Hadrons <i>Total Cross Section; The Running of <math>\alpha_s</math>; Gluon Emission</i>	548
17.3	Deep Inelastic Scattering <i>Deep Inelastic Neutrino Scattering; The Distribution Functions</i>	555
17.4	Hard-Scattering Processes in Hadron Collisions <i>Lepton Pair Production; Kinematics; Jet Pair Production</i>	563
17.5	Parton Evolution <i>The Equivalent Photon Approximation; Multiple Splittings; Evolution Equations for QED; The Altarelli-Parisi Equations</i>	574
17.6	Measurements of $\alpha_s$ Problems	593 595
<b>18</b>	<b>Operator Products and Effective Vertices</b>	<b>599</b>
*18.1	Renormalization of the Quark Mass Parameter	599
*18.2	QCD Renormalization of the Weak Interactions	605
*18.3	The Operator Product Expansion	612
*18.4	Operator Analysis of $e^+e^-$ Annihilation	615
*18.5	Operator Analysis of Deep Inelastic Scattering <i>Kinematics; Expansion of the Operator Product; The Dispersion Integral; Operator Rescaling; Operator Mixing; Relation to the Altarelli-Parisi Equations</i>	621
	Problems	647
<b>19</b>	<b>Perturbation Theory Anomalies</b>	<b>651</b>
*19.1	The Axial Current in Two Dimensions <i>Vacuum Polarization Diagrams; The Current Operator Equation; An Example with Fermion Number Nonconservation</i>	651
*19.2	The Axial Current in Four Dimensions <i>The Current Operator Equation; Triangle Diagrams; Chiral Transformation of the Functional Integral</i>	659
*19.3	Goldstone Bosons and Chiral Symmetries in QCD <i>Spontaneous Breaking of Chiral Symmetry; Anomalies of Chiral Currents</i>	667
*19.4	Chiral Anomalies and Chiral Gauge Theories	676
*19.5	Anomalous Breaking of Scale Invariance Problems	682 686
<b>20</b>	<b>Gauge Theories with Spontaneous Symmetry Breaking</b>	<b>689</b>
20.1	The Higgs Mechanism <i>An Abelian Example; Systematics of the Higgs Mechanism; Non-Abelian Examples; Formal Description</i>	690
20.2	The Glashow-Weinberg-Salam Theory of Weak Interactions <i>Gauge Boson Masses; Coupling to Fermions; Experimental Consequences of the Glashow-Weinberg-Salam Theory; Fermion Mass Terms; The Higgs Boson; A Higgs Sector?</i>	700

\*20.3 Symmetries of the Theory of Quarks and Leptons . . . . . 719  
 Problems . . . . . 728

**21 Quantization of Spontaneously Broken Gauge Theories 731**

21.1 The  $R_\xi$  Gauges . . . . . 732  
*An Abelian Example;  $\xi$  Dependence in Perturbation Theory;  
 Non-Abelian Analysis*

21.2 The Goldstone Boson Equivalence Theorem . . . . . 743  
*Formal Aspects of Goldstone Boson Equivalence;  
 Top Quark Decay;  $e^+e^- \rightarrow W^+W^-$*

\*21.3 One-Loop Corrections in Weak-Interaction Gauge Theory . 758  
*Theoretical Orientation, and a Specific Problem;  
 Influence of Heavy Quark Corrections; Computation of  
 Vacuum Polarization Amplitudes; The Effect of  $m_t$*   
 Problems . . . . . 773

**Final Project: Decays of the Higgs Boson . . . . . 775**

Epilogue

**22 Quantum Field Theory at the Frontier . . . . . 781**

22.1 Strong Strong Interactions . . . . . 782

22.2 Grand Unification and its Paradoxes . . . . . 786

22.3 Exact Solutions in Quantum Field Theory . . . . . 791

22.4 Supersymmetry . . . . . 795

22.5 Toward an Ultimate Theory of Nature . . . . . 798

---

**Appendix: Reference Formulae . . . . . 801**

A.1 Feynman Rules . . . . . 801

A.2 Polarizations of External Particles . . . . . 803

A.3 Numerator Algebra . . . . . 805

A.4 Loop Integrals and Dimensional Regularization . . . . . 806

A.5 Cross Sections and Decay Rates . . . . . 808

A.6 Physical Constants and Conversion Factors . . . . . 809

Bibliography . . . . . 811

Index . . . . . 817

\*These sections may be omitted in a one-year course emphasizing the less formal aspects of elementary particle physics.