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INTRODUCTION TO SUPERSYMMETRY

1.1 The unreasonable effectiveness of the Standard Model

The standard model (SM) of particle physics is well-known to be increasingly obsolete, since it is in accord with all the experimental data. However, the consistency of the model relies on the Higgs field having a vacuum expectation value (VEV) of 246 GeV even though this is highly unstable under quantum loop corrections. This instability can be seen by computing the loop corrections to the Higgs mass term. The fact that these corrections diverge quadratically with the high-energy cutoff is the signal that this instability is a severe problem. Much of the recent interest in supersymmetry (SUSY) has been driven by the possibility that SUSY can cure this instability.

The largest contribution to the Higgs mass correction in the SM of particle physics comes from the top quark loop. The top quark acquires a mass from the VEV $\langle H^0 \rangle$ of the real, neutral component of the Higgs field (denoted by H^0). Given the coupling of the Higgs to the top quark

$$\mathcal{L}_{\text{mass}} = -\frac{y_t}{\sqrt{2}} \bar{t}_L t_R + \text{h.c.} \quad (1.1)$$

(where t_L and t_R are the left-handed and right-handed components of the top quark, y_t is the top Yukawa coupling, and h.c. denotes the hermitian conjugate) and expanding H^0 around its VEV

$$H^0 = \langle H^0 \rangle + h^0 = v + h^0 \quad (1.2)$$

(here h^0 represents the quantum fluctuations around the VEV) we have that the top mass is

$$m_t = \frac{y_t v}{\sqrt{2}} \quad (1.3)$$

Given the coupling in eqn (1.1), we can easily evaluate the Feynman diagram in Fig. 1.1.

The contribution to the Higgs mass squared corresponding to Fig. 1.1 is

$$\delta m_{h^0}^2 = (-1)N_c \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[\frac{-iy_t}{\sqrt{2}} \frac{1}{\not{k} - m_t} \left(\frac{iy_t}{\sqrt{2}} \right) \frac{1}{\not{k} - m_t} \right]$$