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1.1 Tensor Algebra

Before we get to doing the actual calculations, which will also naturally be important to get from the very beginning, we should first review a few basic notions about vector spaces and linear maps (or bilinear forms) defined over vector spaces. We shall assume that the reader is at least familiar with vectors and vector spaces, so we shall try not to go into unnecessary details. Let V denote a vector space of dimension n defined over the set of real numbers \mathbb{R} . Given an arbitrary basis $\{e_i\}_{i=1}^n$, we can write a vector $v \in V$ as

$$v = \sum_{i=1}^n v_i e_i \tag{1.1}$$

One is probably used to see a vector written in the following form

$$v = \sum_{i=1}^n v_i \hat{e}_i \tag{1.2}$$

Here we will suppress the bold vector symbols and adopt the standard Einstein summation convention for repeated indices, so that we get in the compact form (1.1). Let us consider an invertible change of basis given by the matrix A ($\det(A) \neq 0$). We can relate the new basis with the original one by

¹In general, vectors can be defined over \mathbb{C} , but here, we are concerned in this book. One can also define numbers over the p -adic numbers, but this is rather exotic and is not covered in this book.