

---

# Contents

---

|                                                          |     |
|----------------------------------------------------------|-----|
| Preface                                                  | xv  |
| Glossary                                                 | xix |
| <br>                                                     |     |
| I Basics of Robotics                                     | 1   |
| <br>                                                     |     |
| 1 Introduction                                           | 3   |
| 1.1 Mechanics and Control of Robotics . . . . .          | 5   |
| 1.1.1 Basics of Robotics . . . . .                       | 5   |
| 1.1.2 Key Topics . . . . .                               | 6   |
| 1.1.3 Advanced Analysis and Case Studies . . . . .       | 8   |
| 1.2 Robot Architectures . . . . .                        | 9   |
| 1.2.1 Cartesian . . . . .                                | 9   |
| 1.2.2 Cylindrical . . . . .                              | 9   |
| 1.2.3 Spherical . . . . .                                | 11  |
| 1.2.4 SCARA . . . . .                                    | 11  |
| 1.2.5 Articulated . . . . .                              | 12  |
| 1.2.6 Wrists . . . . .                                   | 14  |
| 1.3 Conclusion . . . . .                                 | 15  |
| 1.4 Exercises . . . . .                                  | 15  |
| <br>                                                     |     |
| 2 Planar Kinematics, Velocity, and Statics               | 16  |
| 2.1 Two-Dimensional Space . . . . .                      | 17  |
| 2.2 Transformations . . . . .                            | 17  |
| 2.2.1 Transforming a Vector . . . . .                    | 17  |
| 2.2.2 Transformation Trees . . . . .                     | 19  |
| 2.3 Planar Robot Kinematics . . . . .                    | 20  |
| 2.3.1 Joints . . . . .                                   | 20  |
| 2.3.2 Forward Kinematics . . . . .                       | 21  |
| 2.3.3 Inverse Kinematics . . . . .                       | 24  |
| 2.4 Velocity Analysis . . . . .                          | 26  |
| 2.4.1 Linear and Angular End Effector Velocity . . . . . | 27  |
| 2.4.2 Jacobian . . . . .                                 | 29  |
| 2.4.3 Singularity . . . . .                              | 30  |
| 2.5 Statics Analysis . . . . .                           | 31  |
| 2.6 Workspace . . . . .                                  | 32  |
| 2.7 Conclusion . . . . .                                 | 33  |
| 2.8 Exercises . . . . .                                  | 34  |
| <br>                                                     |     |
| 3 Trajectory Generation                                  | 36  |
| 3.1 Interpolation with Polynomials . . . . .             | 36  |

|           |                                                             |           |
|-----------|-------------------------------------------------------------|-----------|
| 3.2       | Interpolation with Linear and Parabolic Functions . . . . . | 42        |
| 3.3       | Trajectory in Joint Space . . . . .                         | 45        |
| 3.4       | Trajectory in Task Space . . . . .                          | 46        |
| 3.5       | MATLAB® Examples . . . . .                                  | 48        |
| 3.6       | Conclusion . . . . .                                        | 54        |
| 3.7       | Exercises . . . . .                                         | 54        |
| <b>4</b>  | <b>Control Schemes</b>                                      | <b>56</b> |
| 4.1       | Open-Loop Control . . . . .                                 | 56        |
| 4.2       | Closed-Loop Control . . . . .                               | 57        |
| 4.2.1     | Bang-Bang Control . . . . .                                 | 57        |
| 4.2.2     | Proportional Controller . . . . .                           | 59        |
| 4.2.3     | Integral Controller . . . . .                               | 61        |
| 4.2.4     | Derivative Controller . . . . .                             | 61        |
| 4.2.5     | PI and PD Controller . . . . .                              | 62        |
| 4.2.6     | Proportional-Integral-Derivative (PID) Controller . . . . . | 64        |
| 4.3       | Pulse Width Modulation Control . . . . .                    | 65        |
| 4.3.1     | Relation to Joint Control . . . . .                         | 65        |
| 4.4       | Conclusion . . . . .                                        | 66        |
| 4.5       | Exercises . . . . .                                         | 66        |
| <b>II</b> | <b>Key Topics</b>                                           | <b>69</b> |
| <b>5</b>  | <b>General Rotations and Transformations</b>                | <b>71</b> |
| 5.1       | Position and Orientation . . . . .                          | 71        |
| 5.1.1     | Functions of a Rotation Matrix . . . . .                    | 73        |
| 5.2       | General Orientation . . . . .                               | 73        |
| 5.3       | Fixed Angles . . . . .                                      | 75        |
| 5.3.1     | Forward Problem . . . . .                                   | 75        |
| 5.3.2     | Inverse Problem . . . . .                                   | 76        |
| 5.4       | Euler Angles . . . . .                                      | 77        |
| 5.4.1     | Forward Problem . . . . .                                   | 78        |
| 5.4.2     | Inverse Problem . . . . .                                   | 79        |
| 5.5       | General Transformation . . . . .                            | 80        |
| 5.6       | MATLAB® Examples . . . . .                                  | 83        |
| 5.6.1     | Matrix Arithmetic . . . . .                                 | 83        |
| 5.6.2     | Inverse Transformation Matrix . . . . .                     | 84        |
| 5.7       | Conclusion . . . . .                                        | 87        |
| 5.8       | Exercises . . . . .                                         | 88        |
| <b>6</b>  | <b>Forward Kinematics</b>                                   | <b>91</b> |
| 6.1       | Joints . . . . .                                            | 91        |
| 6.2       | Denavit-Hartenberg Notation . . . . .                       | 91        |
| 6.3       | DH Parameters for Prismatic Joints . . . . .                | 97        |
| 6.4       | MATLAB® Examples . . . . .                                  | 102       |
| 6.4.1     | DH Parameter Summary . . . . .                              | 105       |
| 6.5       | Conclusion . . . . .                                        | 106       |
| 6.6       | Exercises . . . . .                                         | 107       |

|           |                                                     |            |
|-----------|-----------------------------------------------------|------------|
| <b>7</b>  | <b>Inverse Kinematics</b>                           | <b>110</b> |
| 7.1       | Basic Techniques . . . . .                          | 110        |
| 7.2       | Analytical Solution to Inverse Kinematics . . . . . | 112        |
| 7.3       | Univariate Polynomial . . . . .                     | 119        |
| 7.4       | Dialytic Method . . . . .                           | 121        |
| 7.5       | Conclusion . . . . .                                | 126        |
| 7.6       | Exercises . . . . .                                 | 127        |
| <b>8</b>  | <b>Jacobian Analysis</b>                            | <b>132</b> |
| 8.1       | Jacobian Matrix . . . . .                           | 132        |
| 8.2       | Velocity in Translation and Rotation . . . . .      | 136        |
| 8.3       | Velocity Propagation . . . . .                      | 140        |
| 8.4       | Statics . . . . .                                   | 144        |
| 8.4.1     | Relation between Jacobians and Effort . . . . .     | 144        |
| 8.5       | Workspace . . . . .                                 | 148        |
| 8.6       | MATLAB <sup>®</sup> Examples . . . . .              | 150        |
| 8.7       | Conclusion . . . . .                                | 154        |
| 8.8       | Exercises . . . . .                                 | 154        |
| <b>9</b>  | <b>Path Planning</b>                                | <b>159</b> |
| 9.1       | Configuration Space . . . . .                       | 159        |
| 9.1.1     | Grid . . . . .                                      | 161        |
| 9.1.2     | Random Sampling . . . . .                           | 162        |
| 9.2       | State Connectivity . . . . .                        | 163        |
| 9.2.1     | Grids . . . . .                                     | 163        |
| 9.2.2     | Random Samples . . . . .                            | 166        |
| 9.2.3     | Connectivity Matrix . . . . .                       | 166        |
| 9.3       | Planning Completeness . . . . .                     | 168        |
| 9.3.1     | Measuring Complexity . . . . .                      | 168        |
| 9.4       | Complete Planning Algorithms . . . . .              | 169        |
| 9.4.1     | First-Search Algorithms . . . . .                   | 169        |
| 9.4.2     | Dijkstra’s Algorithm . . . . .                      | 175        |
| 9.4.3     | A* . . . . .                                        | 179        |
| 9.5       | Sample-Based Planning Algorithms . . . . .          | 184        |
| 9.5.1     | Probabilistic Road Map (PRM) . . . . .              | 184        |
| 9.5.2     | Rapidly Exploring Random Trees (RRT) . . . . .      | 186        |
| 9.6       | Potential Field Planners . . . . .                  | 189        |
| 9.7       | Conclusion . . . . .                                | 189        |
| <b>10</b> | <b>Programming</b>                                  | <b>190</b> |
| 10.1      | Modelling . . . . .                                 | 190        |
| 10.1.1    | Symbolic Functions and Handles . . . . .            | 190        |
| 10.2      | Robot Operating System . . . . .                    | 191        |
| 10.2.1    | Operating Paradigm . . . . .                        | 192        |
| 10.2.2    | ROS Components . . . . .                            | 193        |
| 10.2.3    | ROS System: Case Study . . . . .                    | 195        |
| 10.3      | Conclusion . . . . .                                | 200        |
| <b>11</b> | <b>Lagrangian Dynamics</b>                          | <b>201</b> |
| 11.1      | Rigid Body Dynamics . . . . .                       | 201        |
| 11.2      | Inertia Tensors . . . . .                           | 202        |

|           |                                                                 |            |
|-----------|-----------------------------------------------------------------|------------|
| 11.3      | Principal Moments of Inertia . . . . .                          | 204        |
| 11.3.1    | Parallel Axis Theorem . . . . .                                 | 206        |
| 11.4      | The Lagrangian Method . . . . .                                 | 207        |
| 11.4.1    | Mass Matrix . . . . .                                           | 217        |
| 11.4.2    | Gravity Term . . . . .                                          | 218        |
| 11.4.3    | Friction Term . . . . .                                         | 218        |
| 11.4.4    | Load Term . . . . .                                             | 219        |
| 11.4.5    | MATLAB Example . . . . .                                        | 219        |
| 11.5      | Conclusion . . . . .                                            | 220        |
| 11.6      | Exercises . . . . .                                             | 220        |
| <b>12</b> | <b>Newton-Euler Dynamics</b>                                    | <b>222</b> |
| 12.1      | Newton's and Euler's Equations . . . . .                        | 222        |
| 12.1.1    | Rigid Body Rotation . . . . .                                   | 223        |
| 12.1.2    | Intermediate Axis Theorem . . . . .                             | 225        |
| 12.2      | Outward Propagation . . . . .                                   | 227        |
| 12.3      | Inward Propagation . . . . .                                    | 231        |
| 12.4      | Procedure . . . . .                                             | 233        |
| 12.5      | Twist, Wrench, and $6 \times 6$ Transformation Matrix . . . . . | 236        |
| 12.5.1    | Transformation of Twist . . . . .                               | 236        |
| 12.5.2    | Inverse Transformation of Twist . . . . .                       | 237        |
| 12.5.3    | Transformation of Wrench . . . . .                              | 238        |
| 12.5.4    | Derivative of ${}^0\mathbf{X}_0$ Matrix . . . . .               | 238        |
| 12.5.5    | Physical Interpretation . . . . .                               | 241        |
| 12.6      | Applications in Computing . . . . .                             | 241        |
| 12.6.1    | Computing Efficiency . . . . .                                  | 241        |
| 12.7      | Conclusion . . . . .                                            | 248        |
| 12.8      | Exercises . . . . .                                             | 248        |
| <b>13</b> | <b>Joint Control</b>                                            | <b>252</b> |
| 13.1      | Servo Dynamics . . . . .                                        | 252        |
| 13.2      | Modelling Servos with a Gear Transmission . . . . .             | 255        |
| 13.3      | Fixed Reference Tracking . . . . .                              | 261        |
| 13.3.1    | P Controller . . . . .                                          | 261        |
| 13.3.2    | PD Controller . . . . .                                         | 264        |
| 13.3.3    | PID Controller . . . . .                                        | 269        |
| 13.4      | Error Dynamics . . . . .                                        | 272        |
| 13.4.1    | First-Order Error Dynamics . . . . .                            | 273        |
| 13.4.2    | Second-Order Error Dynamics . . . . .                           | 273        |
| 13.4.3    | Velocity Control . . . . .                                      | 275        |
| 13.5      | Conclusion . . . . .                                            | 276        |
| 13.6      | Exercises . . . . .                                             | 276        |
| <b>14</b> | <b>Computed Torque Control</b>                                  | <b>278</b> |
| 14.1      | SISO Computed Torque Control . . . . .                          | 278        |
| 14.2      | MIMO Computed Torque Control . . . . .                          | 279        |
| 14.3      | Controller with Gravity Compensation . . . . .                  | 282        |
| 14.4      | Lyapunov Stability . . . . .                                    | 283        |
| 14.4.1    | Basic Definition . . . . .                                      | 283        |
| 14.4.2    | Lyapunov's Direct Method . . . . .                              | 285        |
| 14.5      | Dynamic Relation . . . . .                                      | 286        |

|        |                                                                           |            |
|--------|---------------------------------------------------------------------------|------------|
| 14.6   | Stability Analysis of Nonlinear Robotic Controllers . . . . .             | 288        |
| 14.7   | Motion Control in Task Space . . . . .                                    | 289        |
| 14.7.1 | Dynamics in Task Space . . . . .                                          | 290        |
| 14.7.2 | Task Space Dynamics Partitioning . . . . .                                | 291        |
| 14.7.3 | Error Dynamics . . . . .                                                  | 291        |
| 14.8   | Conclusion . . . . .                                                      | 292        |
| 14.9   | Exercises . . . . .                                                       | 293        |
| 15     | <b>Force Control</b>                                                      | <b>295</b> |
| 15.1   | Single Axis Control in Task Space . . . . .                               | 295        |
| 15.2   | Hybrid Motion-Force Control . . . . .                                     | 297        |
| 15.2.1 | Natural Constraints . . . . .                                             | 297        |
| 15.2.2 | Artificial Constraints . . . . .                                          | 298        |
| 15.3   | Impedance Control . . . . .                                               | 300        |
| 15.3.1 | Single Axis Control . . . . .                                             | 300        |
| 15.3.2 | Task Space Control . . . . .                                              | 301        |
| 15.3.3 | Control with Environmental Dynamics . . . . .                             | 301        |
| 15.3.4 | Dynamic Relations . . . . .                                               | 302        |
| 15.4   | Conclusion . . . . .                                                      | 304        |
| 15.5   | Exercises . . . . .                                                       | 304        |
| III    | <b>Advanced Analysis and Case Studies</b>                                 | <b>307</b> |
| 16     | <b>Mobility Analysis</b>                                                  | <b>309</b> |
| 16.1   | Mobility Analysis Based on the Pattern of Transformation Matrix . . . . . | 309        |
| 16.1.1 | Pattern of Transformation Matrix . . . . .                                | 309        |
| 16.1.2 | Case Study—3-UPU Parallel Manipulator . . . . .                           | 311        |
| 16.1.3 | Case Study—3R2T Parallel Manipulator . . . . .                            | 313        |
| 16.1.4 | Case Study—3R1T Parallel Manipulator . . . . .                            | 314        |
| 16.1.5 | Case Study—2-RPS-UPS Parallel Manipulator . . . . .                       | 315        |
| 16.2   | The Order of Mobility . . . . .                                           | 316        |
| 16.2.1 | Local Mobility . . . . .                                                  | 317        |
| 16.2.2 | Case Study—A Novel Spatial Four-Bar Linkage . . . . .                     | 318        |
| 16.3   | Conclusion . . . . .                                                      | 322        |
|        | Bibliography . . . . .                                                    | 322        |
| 17     | <b>Orientation Workspace</b>                                              | <b>324</b> |
| 17.1   | Measurement Principles and Quaternions . . . . .                          | 324        |
| 17.2   | Generalised Volume of Orientation Workspace . . . . .                     | 325        |
| 17.3   | Orientation Workspace Volume of Various Kinematic Descriptions . . . . .  | 327        |
| 17.3.1 | Euler Angles . . . . .                                                    | 327        |
| 17.3.2 | Angle-Axis Representation . . . . .                                       | 328        |
| 17.4   | Case Study . . . . .                                                      | 328        |
| 17.5   | Conclusion . . . . .                                                      | 330        |
|        | Bibliography . . . . .                                                    | 330        |
| 18     | <b>Constraint Analysis for Underactuated Systems</b>                      | <b>332</b> |
| 18.1   | Constrained Minimisation . . . . .                                        | 333        |
| 18.2   | Kinetostatic Constraints . . . . .                                        | 333        |
| 18.2.1 | Generalised Coordinates and Generalised External Forces . . . . .         | 333        |

|        |                                                                                |            |
|--------|--------------------------------------------------------------------------------|------------|
| 18.2.2 | General-Form Kinematic Constraint Equations and Lagrange Multipliers . . . . . | 334        |
| 18.2.3 | Force Constraint Equations . . . . .                                           | 335        |
| 18.2.4 | Hooke Constitutive Equations . . . . .                                         | 335        |
| 18.2.5 | Selection Matrices . . . . .                                                   | 335        |
| 18.2.6 | Array of Unknowns . . . . .                                                    | 336        |
| 18.2.7 | Kinetostatic Constraints . . . . .                                             | 336        |
| 18.3   | Case Study — Soft Robots with Pseudo-Rigid Bodies . . . . .                    | 337        |
| 18.3.1 | 3R-PRB Model of Backbone Sub-Segments . . . . .                                | 338        |
| 18.3.2 | Generalised Coordinates and Generalised External Forces . . . . .              | 338        |
| 18.3.3 | Kinematic Constraint Equations and Lagrange Multipliers . . . . .              | 339        |
| 18.3.4 | Hooke Constitutive Equations . . . . .                                         | 340        |
| 18.3.5 | Constraint Inequalities . . . . .                                              | 341        |
| 18.3.6 | Constrained Minimisation . . . . .                                             | 341        |
| 18.3.7 | Experimental Verification . . . . .                                            | 342        |
| 18.4   | Case Study — The Compliant Five-Link Epicyclic Finger . . . . .                | 343        |
| 18.4.1 | Generalised Coordinates and Generalised External Forces . . . . .              | 344        |
| 18.4.2 | Kinematic Constraint Equations and Lagrange Multipliers . . . . .              | 346        |
| 18.4.3 | Hooke Constitutive Equations . . . . .                                         | 347        |
| 18.4.4 | Force Constraints of Contacts . . . . .                                        | 348        |
| 18.4.5 | Selection Matrices . . . . .                                                   | 348        |
| 18.4.6 | Constrained Minimisation . . . . .                                             | 349        |
| 18.4.7 | Experimental Verification . . . . .                                            | 350        |
| 18.5   | Conclusion . . . . .                                                           | 351        |
|        | Bibliography . . . . .                                                         | 351        |
| 19     | <b>Concentric Tube Robot</b>                                                   | <b>353</b> |
| 19.1   | Overview of Concentric Tube Robot . . . . .                                    | 353        |
| 19.2   | Robot-Independent Mapping . . . . .                                            | 354        |
| 19.3   | Robot-Dependent Mapping . . . . .                                              | 356        |
| 19.3.1 | Constitutive Equation . . . . .                                                | 356        |
| 19.3.2 | Static Equilibrium . . . . .                                                   | 357        |
| 19.3.3 | Variations of the Governing Equation . . . . .                                 | 358        |
| 19.4   | Iteration . . . . .                                                            | 360        |
| 19.5   | Case Studies for Computational Efficiency . . . . .                            | 361        |
| 19.6   | Conclusion . . . . .                                                           | 363        |
|        | Bibliography . . . . .                                                         | 363        |
| 20     | <b>Path Planning of Parallel Manipulators</b>                                  | <b>364</b> |
| 20.1   | Parallel Manipulator Kinematics . . . . .                                      | 364        |
| 20.1.1 | Singularities . . . . .                                                        | 365        |
| 20.1.2 | Example: A 5R Parallel Manipulator . . . . .                                   | 366        |
| 20.2   | The Path Planning Problem . . . . .                                            | 368        |
| 20.3   | Methodology . . . . .                                                          | 369        |
| 20.4   | Generating Charts . . . . .                                                    | 370        |
| 20.4.1 | Singularity Locus . . . . .                                                    | 370        |
| 20.4.2 | Identifying a Chart . . . . .                                                  | 370        |
| 20.4.3 | Constructing a Chart’s $\mathcal{C}$ -space . . . . .                          | 372        |
| 20.5   | Constructing the Atlas . . . . .                                               | 373        |
| 20.6   | Case Study: A 3-RRR Parallel Manipulator . . . . .                             | 374        |
| 20.6.1 | Forward Kinematics . . . . .                                                   | 375        |

|           |                                                                       |            |
|-----------|-----------------------------------------------------------------------|------------|
| 20.6.2    | Continuity Conditions . . . . .                                       | 378        |
| 20.6.3    | Singularity Analysis . . . . .                                        | 378        |
| 20.6.4    | Path Planning . . . . .                                               | 380        |
| 20.6.5    | Performance . . . . .                                                 | 383        |
| 20.7      | Conclusion . . . . .                                                  | 383        |
|           | Bibliography . . . . .                                                | 384        |
| <b>21</b> | <b>Minimally-Invasive Surgical Robot with Remote Centre of Motion</b> | <b>387</b> |
| 21.1      | Minimally Invasive Surgery and MIS Robots . . . . .                   | 387        |
| 21.2      | The Dual-Triangular Linkage . . . . .                                 | 389        |
| 21.2.1    | The Design . . . . .                                                  | 389        |
| 21.2.2    | Proof of Remote Centre of Motion . . . . .                            | 390        |
| 21.2.3    | The Gear-Constrained Dual-Triangular Linkage . . . . .                | 391        |
| 21.2.4    | The 2R1T RCM Mechanism Based on the DT Linkage . . . . .              | 392        |
| 21.3      | The Cable-Constrained Linkage with Remote Centre of Motion . . . . .  | 393        |
| 21.3.1    | The Design . . . . .                                                  | 393        |
| 21.3.2    | Proof of Functioning . . . . .                                        | 394        |
| 21.4      | Advantage of DT/CC Linkage over Parallelogram RCM Mechanism . . . . . | 399        |
| 21.5      | Conclusion . . . . .                                                  | 401        |
|           | Bibliography . . . . .                                                | 401        |
| <b>22</b> | <b>MARS: The Monash Apple Retrieving System</b>                       | <b>402</b> |
| 22.1      | Background and Motivation . . . . .                                   | 402        |
| 22.2      | Virtual Environment Construction . . . . .                            | 403        |
| 22.2.1    | Kinematics . . . . .                                                  | 404        |
| 22.2.2    | Auxiliary Frame Assignments . . . . .                                 | 406        |
| 22.3      | Pose Optimisation . . . . .                                           | 406        |
| 22.3.1    | Objective Function . . . . .                                          | 408        |
| 22.3.2    | Kinematic Constraints . . . . .                                       | 408        |
| 22.3.3    | Collision Constraints . . . . .                                       | 409        |
| 22.3.4    | Kinematic Configuration Constraint . . . . .                          | 410        |
| 22.4      | Path Planning . . . . .                                               | 410        |
| 22.4.1    | Planning Algorithm . . . . .                                          | 410        |
| 22.4.2    | Harvesting Sequence . . . . .                                         | 411        |
| 22.5      | Results . . . . .                                                     | 415        |
| 22.5.1    | Harvest Success Rate . . . . .                                        | 415        |
| 22.5.2    | Cycle Time . . . . .                                                  | 417        |
| 22.6      | Conclusion . . . . .                                                  | 417        |
|           | Bibliography . . . . .                                                | 418        |
| <b>IV</b> | <b>Appendix</b>                                                       | <b>421</b> |
| <b>23</b> | <b>Appendices for Chapter 5</b>                                       | <b>423</b> |
| 23.1      | Angle-Axis . . . . .                                                  | 423        |
| 23.1.1    | Forward Problem . . . . .                                             | 423        |
| 23.1.2    | Inverse Problem . . . . .                                             | 424        |
| 23.2      | Quaternion . . . . .                                                  | 425        |
| 23.3      | Alternate Expression of Rodrigues' Formula . . . . .                  | 427        |

|                                                                                               |            |
|-----------------------------------------------------------------------------------------------|------------|
| <b>24 Appendices for Chapter 8</b>                                                            | <b>428</b> |
| 24.1 Proof of $\mathbf{Q} \mathbf{Q}^T$ Being Screw-Symmetric . . . . .                       | 428        |
| 24.2 Proof of a Screw-Symmetric Matrix Being the Crossproduct Matrix of a<br>Vector . . . . . | 428        |
| 24.3 Proof of $\omega$ . . . . .                                                              | 429        |
| <b>25 Appendices for Chapter 11</b>                                                           | <b>430</b> |
| 25.1 Inertia Tensor . . . . .                                                                 | 430        |
| 25.2 Euler's Law . . . . .                                                                    | 430        |
| 25.3 Proof of Parallel-Axis Theorem . . . . .                                                 | 431        |
| <b>Index</b>                                                                                  | <b>433</b> |