

As experimental data sets have grown and computational power has increased, new tools have been developed that have the power to model new systems and fundamentally alter how current systems are analyzed. This book brings together modern computational tools to provide an accurate understanding of dynamic data. The techniques build on pencil-and-paper mathematical techniques that go back decades and sometimes even centuries. The result is an introduction to state-of-the-art methods that complement, rather than replace, traditional analysis of time-dependent systems.

*Data-Driven Methods for Dynamic Systems* provides readers with

- methods not found in other texts as well as novel ones developed just for this book;
- an example-driven presentation that provides background material and descriptions of methods without getting bogged down in technicalities;
- examples that demonstrate the applicability of a method and introduce the features and drawbacks of their application; and
- a code repository in the online supplementary material that can be used to reproduce every example and that can be repurposed to fit a variety of applications not found in the book.

### Audience

This book is intended as an introduction to the field of data-driven methods for graduate students. It will also be of interest to researchers who want to familiarize themselves with the discipline. It can be used in courses on dynamical systems, differential equations, and data science.

### About the Author



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