

Contents

Preface	xiii
Introduction	1
Part I 1-D MHD in Ten Weeks	5
1 The Fundamentals of Hydrodynamics	7
1.1 Definition of a fluid	7
1.2 A quick review of kinetic theory	9
1.3 The equations of ideal hydrodynamics	12
1.4 The internal energy density	18
1.5 Primitive, integral, and conservative form	20
Problem Set 1	22
2 Selected Applications of Hydrodynamics	26
2.1 Sound waves	26
2.1.1 Wave equation approach	26
2.1.2 Eigenvalue approach	29
2.2 Rankine–Hugoniot jump conditions	33
2.2.1 Case 1: Trivial solution	35
2.2.2 Case 2: Tangential discontinuity	35
2.2.3 Case 3: Shock	36
2.3 Bores and hydraulic jumps (optional)	42
2.3.1 Bores in the “lab frame”	46
2.4 Bernoulli’s theorem	47
Problem Set 2	56
3 The Hydrodynamical Riemann Problem	69
3.1 Eulerian and Lagrangian frames of reference	70
3.2 The three characteristics of hydrodynamics	72
3.3 Characteristic paths and space-time diagrams	74
3.4 The MoC and the Riemann problem	78
3.5 Non-linear hydrodynamical waves	81
3.5.1 Hyperbolic system of equations	81

3.5.2	Left and right eigenvectors (optional)	82
3.5.3	Hydrodynamical rarefaction fans	85
3.6	Solution to the HD Riemann problem	90
	Problem Set 3	94
4	The Fundamentals of Magnetohydrodynamics	99
4.1	A brief introduction to MHD	99
4.2	The ideal induction equation	101
4.3	Alfvén's theorem	103
4.4	Modifications to the momentum equation	106
4.5	The MHD Poynting power density	107
4.6	Modifications to the total energy equation	107
4.7	The equations of ideal MHD	109
4.8	Vector potential and magnetic helicity (optional)	112
4.8.1	Magnetic topology	114
	Problem Set 4	117
5	MHD Waves and Discontinuities	123
5.1	Primitive and conservative equations of MHD	124
5.2	MHD wave families	127
5.2.1	(Shear) Alfvén waves	130
5.2.2	Fast and slow magnetosonic waves	139
5.2.3	Summary of MHD waves	146
5.3	The MHD Rankine–Hugoniot jump conditions	149
5.3.1	Case 1: Trivial solution	154
5.3.2	Case 2: Contact discontinuity	154
5.3.3	Case 3: Tangential discontinuity	154
5.3.4	Case 4: Rotational discontinuity	155
5.3.5	Case 5: Fast, slow, and intermediate shocks	155
	Problem Set 5	172
6	The MHD Riemann Problem	183
6.1	Overview	183
6.2	Non-linear MHD waves	185
6.2.1	Fast and slow eigenkets	186
6.2.2	Fast and slow rarefaction fans	192
6.3	Space-time diagrams	202
6.4	An MHD Riemann solver	204
6.4.1	Problem parameters	205
6.4.2	Strategy for the Riemann solver	208
6.4.3	Algorithm for an exact MHD Riemann solver	210
6.4.4	Sample problems	221
	Problem Set 6	235

Part II Additional Topics in (M)HD	241
7 Fluid Instabilities	243
7.1 Kelvin–Helmholtz instability	245
7.1.1 Normal mode analysis of the KHI	246
7.1.2 The development of the KHI	251
7.1.3 The KHI in nature	252
7.1.4 Numerical analysis of the KHI (optional)	254
7.2 Rayleigh–Taylor instability	256
7.2.1 Normal mode analysis of the RTI	257
7.2.2 Numerical analysis of the RTI (optional)	262
7.2.3 Kruskal–Schwarzschild instability	267
7.3 Magneto-rotational instability	268
7.3.1 Mathematical model of the MRI	269
7.3.2 Physical model of the MRI	275
7.3.3 Angular momentum transport	278
7.3.4 Numerical analysis of the MRI (optional)	283
7.4 Parker instability	286
7.4.1 A qualitative description	286
7.4.2 A quantitative description (optional)	291
Problem Set 7	305
8 Viscid Hydrodynamics	309
8.1 Introduction	309
8.2 The stress tensor	310
8.2.1 The trace of the stress tensor	312
8.2.2 Viscosity and Newtonian fluids	314
8.2.3 Non-isotropic “pressure”	315
8.3 The Navier–Stokes equation	317
8.4 The viscid energy equation	320
8.4.1 Viscous dissipation	321
8.5 The Reynolds number	322
8.6 Applications	326
8.6.1 Plane laminar viscous flow	327
8.6.2 Forced flow between parallel plates	328
8.6.3 Open channel flow	329
8.6.4 Hagen–Poiseuille flow	333
8.6.5 Couette flow	334
Problem Set 8	337
9 Steady-State MHD	341
9.1 The Weber–Davis constants	342
9.2 The MHD Bernoulli function	349
9.2.1 Critical launching angle	354

9.3	Stellar winds	357
9.3.1	Critical points	359
9.3.2	The Weber–Davis Model	361
9.4	Astrophysical jets (optional)	366
	Problem Set 9	371
10	Non-ideal MHD	378
10.1	Introducing non-ideal MHD	378
10.2	The three players	380
10.2.1	A weakly ionised, isothermal, one-fluid model	380
10.2.2	Relative importance of the non-ideal terms	387
10.3	Resistive dissipation	391
10.3.1	The resistive induction equation	391
10.3.2	Dissipation of magnetic energy	392
10.3.3	Magnetic diffusion and reconnection	393
10.3.4	Dynamo theory (optional)	399
10.4	The Hall effect	406
10.4.1	The case of a completely ionised fluid	408
10.4.2	Magnetic reconnection, revisited	411
10.5	Ambipolar diffusion	414
10.5.1	Overview and motivation	414
10.5.2	A two-fluid, non-isothermal model for AD	416
	Problem Set 10	428
	Appendices	441
A	Essentials of Vector Calculus	443
A.1	Vector identities	443
A.1.1	Identities involving dyadics	444
A.1.2	Vector derivatives of $\vec{r}$	445
A.2	Theorems of vector calculus	445
A.3	Orthogonal coordinate systems	446
A.4	Euler’s and the momentum equations	448
A.5	The Lorentz force	451
B	Essentials of Electrodynamics	453
B.1	Maxwell’s equations	453
B.2	Electric energy density	454
B.3	Magnetic energy density	455
B.4	Resistive energy density	457
B.5	The Poynting vector	458
C	The “Conics” of PDEs	459

D The Secant Method	462
D.1 Univariate root finder	462
D.2 Multivariate root finder	465
E Roots of a Cubic	467
F Sixth-Order Runge–Kutta	468
G Coriolis' Theorem	476
H The Diffusion Equation	484
Variable Glossary	486
References	493
Index	500

Thus, MHD has not had as long a history as other branches of physics in which textbooks could accumulate, particularly at the undergraduate level. Indeed, MHD has largely been considered a graduate-level subject and, because of this, the vast majority of existing texts specialise in areas such as fusion physics, solar physics, and planetary discs, many written for students already with some familiarity of plasma physics or fluid dynamics.

Another extenuating circumstance is MHD is a divided field whose practitioners – largely plasma physicists and fluid dynamicists – approach the subject in two very different ways. From a plasma physicist's point of view (PoV), an MHD system is the isotropic limit of an ensemble of charged particles – a plasma – governed by velocity moments of the *Vlasov–Boltzmann equation*, a 6-D inhomogeneous partial differential equation (PDE) at the heart of plasma physics. From a fluid dynamicist's PoV, an MHD system is never considered as an ensemble of particles, but rather as a continuous medium governed by simple conservation rules that *any* undergraduate physics student can understand. This leads to a hyperbolic set of equations that can be analysed entirely in terms of waves. The two approaches couldn't be more different.

After thirty years of teaching graduate and undergraduate (astro)physics, it is my considered opinion that for MHD to be approachable by undergraduates, it needs to be taught from the fluids PoV. Wave mechanics – so fundamental in classical mechanics, electrodynamics, and quantum mechanics – is already ingrained in the mind of a fourth-year student. On the other hand, velocity moments of a six-

¹On pages 127–128, there's an amusing anecdote on what – or who – changed the physics community's collective mind on MHD, and a link to Anthony Peratt's short biography on Alfvén.