

CONTENTS

12 INTRODUCTION

ANCIENT AND CLASSICAL PERIODS

6000 BCE—500 CE



22 Numerals take their places
Positional numbers

28 The square as the highest power
Quadratic equations

32 The accurate reckoning for inquiring into all things
The Rhind papyrus

34 The sum is the same in every direction
Magic squares

36 Number is the cause of gods and daemons
Pythagoras

44 A real number that is not rational
Irrational numbers

46 The quickest runner can never overtake the slowest
Zeno's paradoxes of motion

48 Their combinations give rise to endless complexities
The Platonic solids

50 Demonstrative knowledge must rest on necessary basic truths
Syllogistic logic

52 The whole is greater than the part
Euclid's *Elements*

58 Counting without numbers
The abacus



60 Exploring pi is like exploring the Universe
Calculating pi

66 We separate the numbers as if by some sieve
Eratosthenes' sieve

68 A geometrical tour de force
Conic sections

70 The art of measuring triangles
Trigonometry

76 Numbers can be less than nothing
Negative numbers

80 The very flower of arithmetic
Diophantine equations

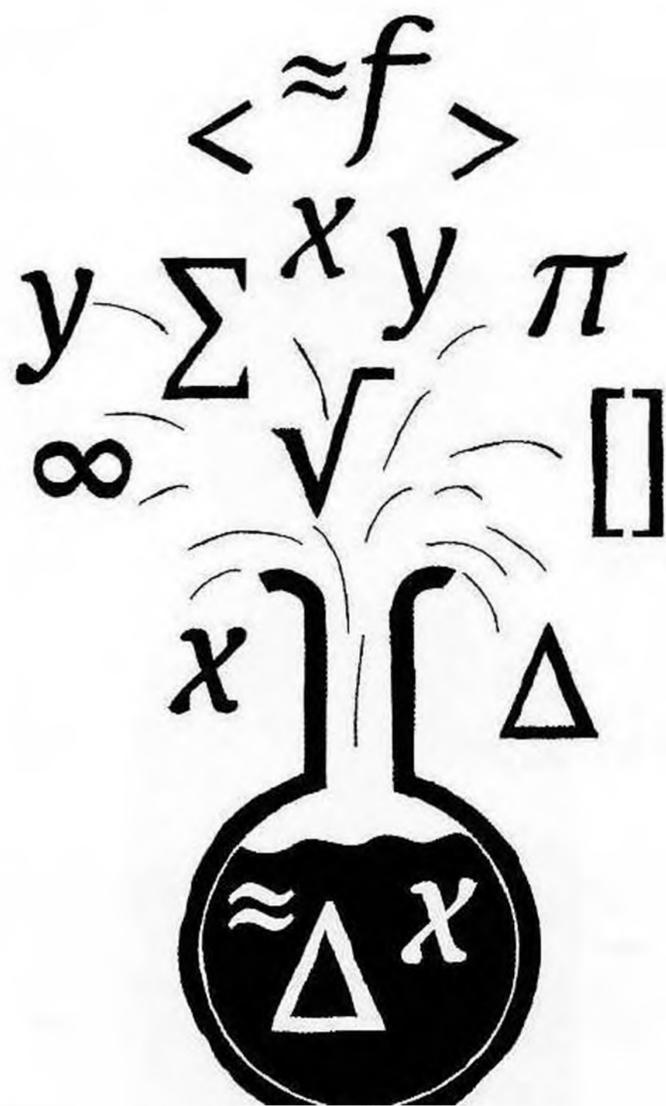
82 An incomparable star in the firmament of wisdom
Hypatia

83 The closest approximation of pi for a millennium
Zu Chongzhi

THE MIDDLE AGES

500–1500

- 88 **A fortune subtracted from zero is a debt**
Zero
- 92 **Algebra is a scientific art**
Algebra
- 100 **Freeing algebra from the constraints of geometry**
The binomial theorem
- 102 **Fourteen forms with all their branches and cases**
Cubic equations
- 106 **The ubiquitous music of the spheres**
The Fibonacci sequence
- 112 **The power of doubling**
Wheat on a chessboard



THE RENAISSANCE

1500–1680

- 118 **The geometry of art and life**
The golden ratio
- 124 **Like a large diamond**
Mersenne primes
- 125 **Sailing on a rhumb**
Rhumb lines
- 126 **A pair of equal-length lines**
The equals sign and other symbology
- 128 **Plus of minus times plus of minus makes minus**
Imaginary and complex numbers
- 132 **The art of tenths**
Decimals
- 138 **Transforming multiplication into addition**
Logarithms
- 142 **Nature uses as little as possible of anything**
The problem of maxima
- 144 **The fly on the ceiling**
Coordinates
- 152 **A device of marvelous invention**
The area under a cycloid
- 154 **Three dimensions made by two**
Projective geometry
- 156 **Symmetry is what we see at a glance**
Pascal's triangle
- 162 **Chance is bridled and governed by law**
Probability
- 166 **The sum of the distance equals the altitude**
Viviani's triangle theorem
- 167 **The swing of a pendulum**
Huygens's tautochrone curve



168 With calculus I can predict the future
Calculus

176 The perfection of the science of numbers
Binary numbers

THE ENLIGHTENMENT

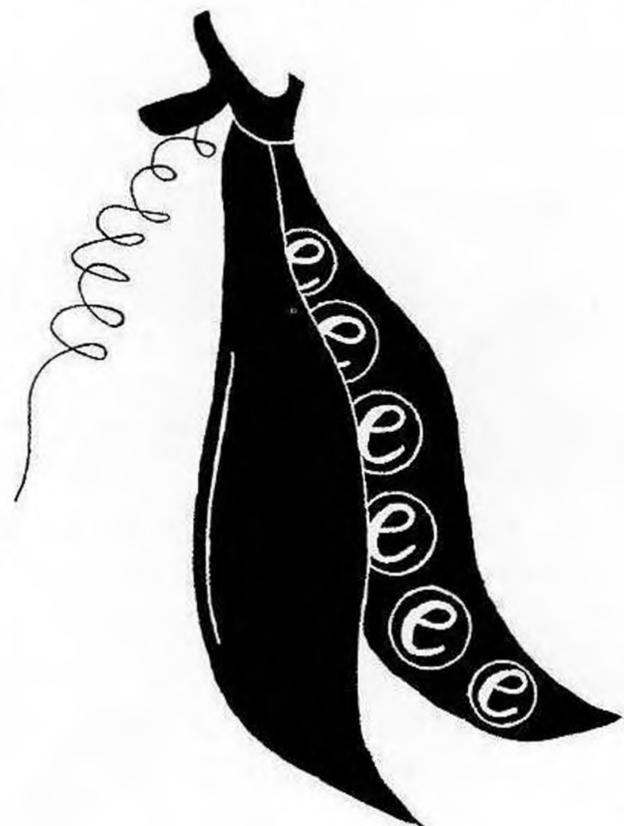
1680–1800

182 To every action there is an equal and opposite reaction
Newton's laws of motion

184 Empirical and expected results are the same
The law of large numbers

186 One of those strange numbers that are creatures of their own
Euler's number

192 Random variation makes a pattern
Normal distribution



194 The seven bridges of Königsberg
Graph theory

196 Every even integer is the sum of two primes
The Goldbach conjecture

197 The most beautiful equation
Euler's identity

198 No theory is perfect
Bayes' theorem

200 Simply a question of algebra
The algebraic resolution of equations

202 Let us gather facts
Buffon's needle experiment

204 Algebra often gives more than is asked of her
The fundamental theorem of algebra

THE 19TH CENTURY

1800–1900

214 Complex numbers are coordinates on a plane
The complex plane

216 Nature is the most fertile source of mathematical discoveries
Fourier analysis

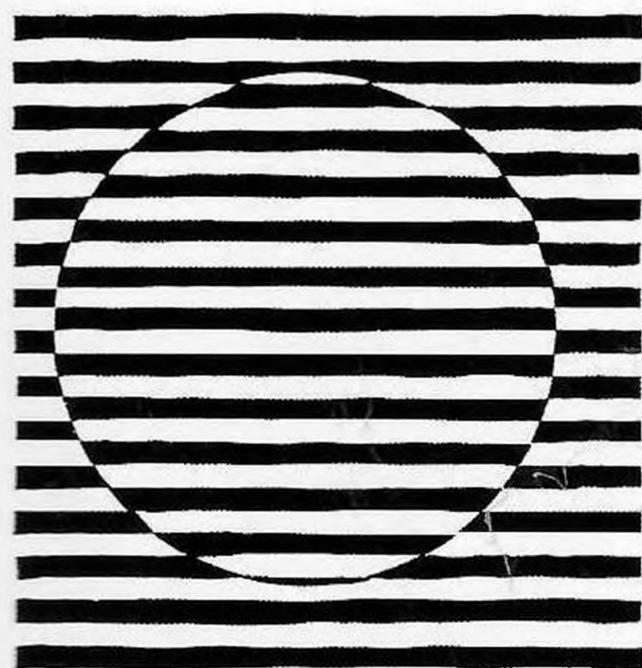
218 The imp that knows the positions of every particle in the Universe
Laplace's demon

220 What are the chances?
The Poisson distribution

221 An indispensable tool in applied mathematics
Bessel functions

222 It will guide the future course of science
The mechanical computer





226 A new kind of function
Elliptic functions

228 I have created another world out of nothing
Non-Euclidean geometries

230 Algebraic structures have symmetries
Group theory

234 Just like a pocket map
Quaternions

236 Powers of natural numbers are almost never consecutive
Catalan's conjecture

238 The matrix is everywhere
Matrices

242 An investigation into the laws of thought
Boolean algebra

248 A shape with just one side
The Möbius strip

250 The music of the primes
The Riemann hypothesis

252 Some infinities are bigger than others
Transfinite numbers

254 A diagrammatic representation of reasonings
Venn diagrams

255 The tower will fall and the world will end
The Tower of Hanoi

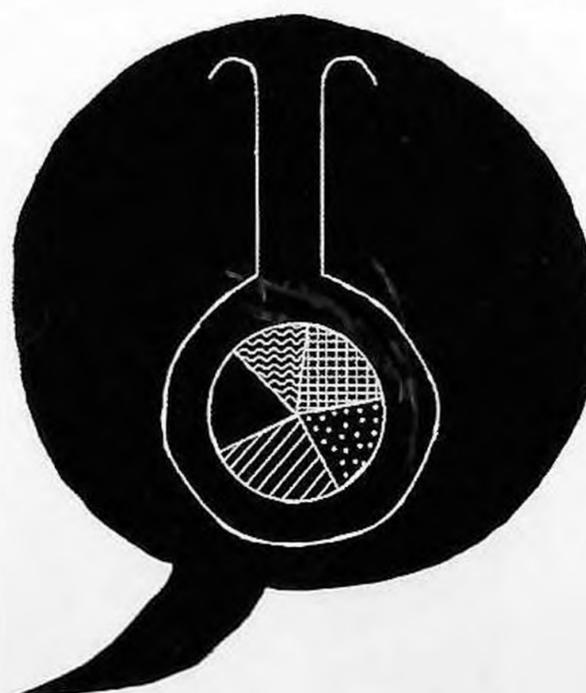
256 Size and shape do not matter, only connections
Topology

260 Lost in that silent, measured space
The prime number theorem

MODERN MATHEMATICS 1900–PRESENT

266 The veil behind which the future lies hidden
23 problems for the 20th century

268 Statistics is the grammar of science
The birth of modern statistics



272 A freer logic emancipates us
The logic of mathematics

274 The Universe is four-dimensional
Minkowski space

276 Rather a dull number
Taxicab numbers

278 A million monkeys banging on a million typewriters
The infinite monkey theorem

280 She changed the face of algebra
Emmy Noether and abstract algebra

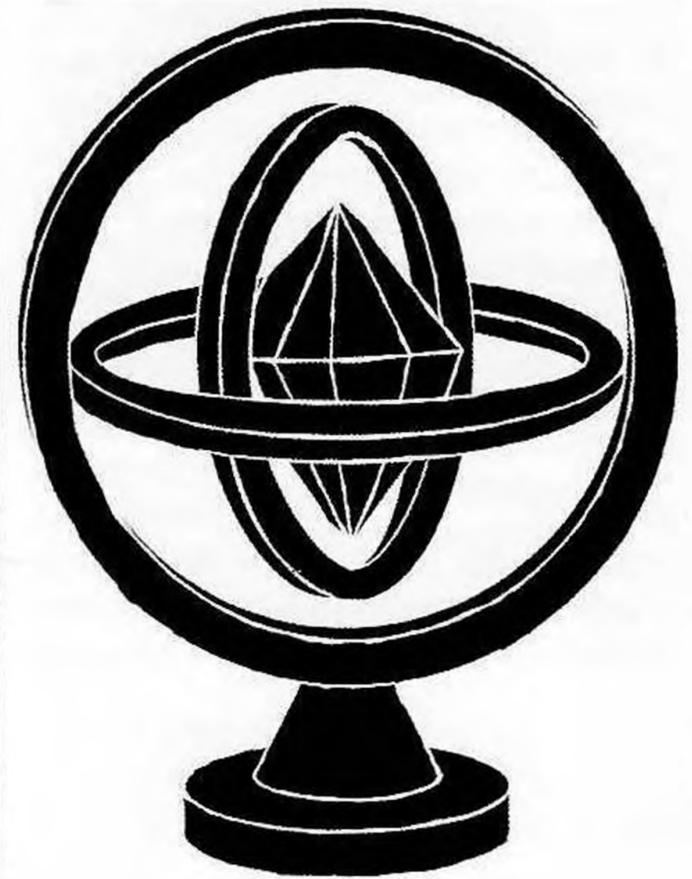
282 Structures are the weapons of the mathematician
The Bourbaki group

284 A single machine to compute any computable sequence
The Turing machine



- 290 Small things are more numerous than large things**
Benford's law
- 291 A blueprint for the digital age**
Information theory
- 292 We are all just six steps away from each other**
Six degrees of separation
- 294 A small positive vibration can change the entire cosmos**
The butterfly effect
- 300 Logically things can only partly be true**
Fuzzy logic
- 302 A grand unifying theory of mathematics**
The Langlands Program
- 304 Another roof, another proof**
Social mathematics

- 305 Pentagons are just nice to look at**
The Penrose tile
- 306 Endless variety and unlimited complication**
Fractals
- 312 Four colors but no more**
The four-color theorem
- 314 Securing data with a one-way calculation**
Cryptography
- 318 Jewels strung on an as-yet invisible thread**
Finite simple groups
- 320 A truly marvelous proof**
Proving Fermat's last theorem
- 324 No other recognition is needed**
Proving the Poincaré conjecture



326 DIRECTORY

336 GLOSSARY

344 INDEX

351 QUOTATIONS

352 ACKNOWLEDGMENTS