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The modern world has witnessed remarkable applications in the dynamic field of image processing, where operations transform an image to enhance it or extract vital information. It is a vibrant and diverse field encompassing various applications, such as facial recognition, image segmentation and compression, noise reduction, and more. These applications require sophisticated techniques to transform, enhance, and extract image information. However, these techniques also demand substantial computational resources for image storage and processing, which pose significant challenges for scalability and efficiency. Therefore, there is a critical need for more advanced and innovative methods to handle visual information. On the other hand, quantum computing defines a probabilistic approach to represent classical information using methods from quantum theory. Quantum computing offers a probabilistic and parallel approach to computation, which differs fundamentally from the deterministic and sequential approach of classical computing. The basic unit of quantum information, the qubit, can exist in a superposition of two states until measured, which enables quantum parallelism and entanglement. These quantum phenomena can provide exponential speedups and enhanced security for specific computational tasks, such as factoring large numbers, searching unsorted databases, simulating quantum systems, and solving linear systems of equations.

Quantum image processing (QIP) is a research branch of quantum information and computing that aims to exploit the advantages of quantum computing for image processing. QIP studies how to encode and process images using various quantum image representations and operations in a quantum computer. QIP has the potential to outperform classical image processing in terms of computing speed, security, and minimum storage requirement. However, QIP also faces many challenges and open questions, such as quantum superiority, reading the classical data, measurement, noise and error mitigation, scalability and compatibility, and the practical implementation of QIP algorithms and circuits.

In this book, we provide a comprehensive introduction to QIP, covering the theoretical foundations, recent technological developments, quantum image encoding, and practical QIP applications. We describe the existing quantum image representations and their operations, such as geometric transformations, color transformations, filtering, and enhancement. We also explore the emerging topics and applications of QIP, such as quantum image filtration in the frequency domain, convolution, and fast unitary transforms. We discuss the current state of QIP research, addressing the controversies and opportunities, as well as the challenges and future directions of QIP. We illustrate the QIP algorithms and circuits with detailed examples, diagrams, and code snippets using the Qiskit framework. We also provide exercises and references for further learning and research.