

CAMBRIDGE TRACTS  
IN THEORETICAL  
COMPUTER SCIENCE

Introducing Stone–Priestley duality theory and its applications to logic and theoretical computer science, this book equips graduate students and researchers with the theoretical background necessary for reading and understanding current research in the area.

After giving a thorough introduction to the algebraic, topological, logical, and categorical aspects of the theory, the book covers two advanced applications in computer science, namely in domain theory and automata theory. These topics are at the forefront of active research seeking to unify semantic methods with more algorithmic topics in finite model theory. Frequent exercises punctuate the text, with hints and references provided.

“This book introduces efficiently Stone–Priestley duality theory for bounded distributive lattices, thereby laying solid mathematical foundations for applications in mathematics and computer science. Readers interested in the fields of domain theory and automata theory will see the general duality theory bearing fruit and opening doors to further applications.”

Jorge Almeida, *Universidade do Porto*

“This book is a textbook and also a research monograph. For undergraduates, there is the basic duality; for postgraduates, applications in algebra, topology, and logic, and to theoretical computer science. Then, there are research themes to develop. The applications to CS are exciting and not published as a book before.”

Mirna Džamonja, *IRIF, CNRS-Université de Paris*



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