

Contents

Preface.....ix

About the Authors.....xi

Chapter 1 Double Integration..... 1

1.1 Introduction 1

1.2 Double Integral over Rectangular Regions.....3

1.2.1 Exercise 7

1.3 Double Integrals over Non-Rectangular Regions..... 8

1.4 Reversing the Order of Integration 15

1.4.1 Exercises 18

1.5 Double Integration over Polar Rectangular Regions 19

1.6 Double Integration over General Polar Regions25

1.6.1 Exercises29

1.7 Engineering Applications30

1.7.1 Density Distributions of a Two-Dimensional Thin Layer 30

1.7.2 Centre of Mass of a Two-Dimensional Thin Layer (Lamina) 33

1.7.3 Moment of Inertia of a Two-Dimensional Thin Layer (Lamina) 36

1.7.4 Exercises 37

Chapter 2 Ordinary Differential Equations 39

2.1 Introduction 39

2.2 Vibrating Spring 39

2.3 ODEs – The Basics 42

2.4 Separable ODEs.....44

2.4.1 Exercises 48

2.5 Linear Equations..... 49

2.6 First-Order Linear Equations..... 50

2.6.1 Exercises 55

2.7 Second-Order Linear ODEs with Constant Coefficients..... 56

2.7.1 Homogeneous Equations with Constant Coefficients ... 57

2.7.2 Exercises 63

2.8 Homogeneous Equations, the Initial Value Problems 63

2.8.1 Exercise 66

2.9 Non-Homogeneous Equations 67

2.9.1 The Method of Undetermined Coefficients..... 67

2.9.2 Exercises 73

2.10 Numerical Solutions of Differential Equations 75

2.10.1 Euler Method 75

2.10.2	Implementation of the Euler Method Using Excel	78
2.10.3	Exercises	81
2.11	Engineering Applications	82
2.11.1	Falling Objects	82
2.11.2	Mixture of Solutions	85
2.11.3	Vibration of a Spring-Mass System	87
2.11.4	Fluid Flow Streamlines	93
2.11.5	Heat Transfer – Fourier’s Law of Heat Conduction	93
2.11.6	Electrical Circuit	95
2.11.7	Exercises	98
Chapter 3	Laplace Transform	101
3.1	Introduction	101
3.2	Definition of the Laplace Transform	102
3.3	Linear Property of the Laplace Transform	102
3.4	Inverse Laplace Transform	104
3.5	The First Shifting Theorem	106
3.5.1	Exercises	107
3.6	Inverse Laplace Transform Using Completing the Square	108
3.6.1	Exercises	109
3.7	Derivatives and the Laplace Transform	110
3.7.1	Using Laplace Transform to Solve ODEs	111
3.7.2	Exercises	114
3.8	The Unit Step Function	114
3.8.1	Products Involving Unit Step Functions	116
3.8.2	Laplace Transform of the Unit Step Function	119
3.9	The Second Shifting Theorem	120
3.9.1	Exercises	123
3.10	Dirac Delta Function	129
3.10.1	Exercises	134
3.11	Convolution	137
3.11.1	Visual Explanation	138
3.11.2	Some Useful Property of Convolution	144
3.11.3	Laplace Transform of $f * g$	145
3.11.4	Exercises	145
3.12	Application in Control Engineering	148
3.13	Table of Laplace Transforms	150
Chapter 4	Linear Systems of Differential Equations	153
4.1	Introduction	153
4.2	Eigenvalues	154
4.3	Eigenvectors	155
4.3.1	Exercises	160
4.4	Repeated Eigenvalues	161

4.5	Complex Eigenvalues and Associated Eigenvectors	162
4.5.1	Exercises	168
4.6	Diagonalisation of a Matrix	169
4.6.1	Exercises	172
4.7	Homogeneous Linear Systems	173
4.7.1	Solving 2×2 Systems (Distinct Real Eigenvalues)	174
4.7.2	Solving 2×2 Systems (Algebraic Multiplicity 2, Geometric Multiplicity 1)	177
4.7.3	Solving 2×2 Second-Order Systems	181
4.7.4	Solving 3×3 Systems (3 Distinct Real Eigenvalues)	184
4.7.5	Solving 3×3 Systems (Algebraic Multiplicity 2 and Geometric Multiplicity 2)	185
4.7.6	Solving 3×3 Systems (Algebraic Multiplicity 2 and Geometric Multiplicity 1)	187
4.7.7	Solving 3×3 Systems (Algebraic Multiplicity 3 and Geometric Multiplicity 2)	190
4.7.8	Solving 3×3 Systems (Algebraic Multiplicity 3 and Geometric Multiplicity 1)	196
4.7.9	Exercises	199
4.8	Systems with Complex Eigenvalues	203
4.8.1	Exercises	208
4.9	Engineering Applications	208
4.9.1	A Two-Tank Mixing Problem	209
4.9.2	A Three-Tank Mixing Problem	210
4.9.3	Modelling 2 Degrees-of-Freedom Spring-Mass-Damper System	215

Chapter 5	Fourier Series	217
5.1	Introduction	217
5.2	Periodic Functions	217
5.3	Adding Periodic Signals with Unequal Frequencies	219
5.4	Adding Signals When Frequencies Are Integer Multiple of Smallest Frequency	225
5.5	The Beat Phenomenon	226
5.6	Frequency Domain (Frequency Spectrum)	229
5.6.1	Exercises	231
5.7	Fourier Series of the Square Wave	233
5.8	Fourier Series of Periodic Functions	236
5.9	Frequency Spectrum of the Fourier Series	238
5.9.1	Exercises	249
5.10	Engineering Applications	255
5.10.1	Vibration Analysis	255
5.10.2	Voltage Output of a Rectifier	257
5.10.3	Exercises	258

Chapter 6	Statistics	260
6.1	Introduction	260
6.2	Population	261
6.3	Sample	261
6.4	Random Variable	261
6.5	Qualitative Variables	261
6.6	Quantitative Variables	261
6.7	Descriptive Statistics	262
6.7.1	Summarising Data	262
6.7.2	Frequency Distribution (Discrete Variable)	262
6.8	The Mean, Standard Deviation and Median	266
6.8.1	Exercises	267
6.9	Continuous Quantitative Variables and Probability Distribution	271
6.10	Normal Distribution	274
6.10.1	Standard Normal Distribution	276
6.10.2	Exercises	279
6.10.3	Standardising the Random Variable X	279
6.10.4	Exercises	281
6.10.5	Calculating the X -Value by Knowing the Probability	282
6.10.6	Exercises	285
6.11	Estimation and Confidence Intervals	286
6.11.1	Point Estimation	286
6.11.2	Distribution of the Sample Mean	287
6.11.3	Exercises	289
6.11.4	Interval Estimates	289
6.11.5	Exercises	292
6.11.6	The t -Distribution	293
6.11.7	Exercises	296
6.12	Hypothesis Testing	297
6.12.1	Exercises	300
6.13	Correlation and Regression	301
6.13.1	Introduction	301
6.13.2	Correlation	303
6.13.3	Regression	304
6.13.4	Exercises	306
Index		311