

Discrete Structures

The aim of this text is to introduce discrete mathematics to beginning students of mathematics or computer science. It does this by bringing some coherency into the seemingly incongruent subjects that compose discrete math, such as logic, set theory, algebra, and combinatorics. It emphasizes their theoretical foundations and illustrates proofs along the way. The book prepares readers for the analysis of algorithms by discussing asymptotic analysis and a discrete calculus for sums. The book also deduces combinatorial methods from the foundations that are laid out. Unlike other texts on this subject, there is a greater emphasis on foundational material that leads to a better understanding. To further assist the reader in grasping and practicing concepts, roughly 690 exercises are provided at various levels of difficulty. Readers are encouraged to study the examples in the text and solve as many of the exercises as possible.

The text is intended for freshman or sophomore undergraduate students in mathematics, computer science, or similar majors. The assumed background is precalculus. The chapter dependency chart included is designed to help students, independent readers, and instructors follow a systematic path for learning and teaching the material, with the option to explore material in later chapters.

ISBN 978-3-031-73433-5



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