

This standard textbook on modern graph theory combines the authority of a classic with the engaging freshness of style that is the hallmark of active mathematics. It covers the core material of the subject, with concise yet complete proofs, while offering glimpses of more advanced methods in each field via one or two deeper results.

This is a major new edition. Among many other improvements, it offers additional tools for applying the regularity lemma, brings the tangle theory of graph minors up to the cutting edge of current research, and addresses new topics such as chi-boundedness in perfect graph theory.

The book can be used as a reliable text for an introductory graduate course and is also suitable for self-study.

From the reviews:

“Deep, clear, wonderful. This is a serious book about the heart of graph theory. It has depth and integrity.” *Persi Diaconis & Ron Graham, SIAM Review*

“The book has received a very enthusiastic reception, which it amply deserves. A masterly elucidation of modern graph theory.” *Bulletin of the Institute of Combinatorics and its Applications*

“Succeeds dramatically ... a hell of a good book.” *MAA Reviews*

“ ... like listening to someone explain mathematics.” *Bulletin of the AMS*

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