CONTENTS

Chapter I Simplicial sets 1
1. Basic definitions 3
2. Realization 7
3. Kan complexes 9
4. Anodyne extensions 14
5. Function complexes 20
6. Simplicial homotopy 23
7. Simplicial homotopy groups 25
8. Fundamental groupoid 36
9. Categories of fibrant objects 40
10. Minimal fibrations 47
11. The closed model structure 59

Chapter II Model Categories 65
1. Homotopical algebra 66
2. Simplicial categories 81
3. Simplicial model categories 89
4. The existence of simplicial model category structures 97
5. Examples of simplicial model categories 101
6. A generalization of Theorem 4.1 109
7. Quillen's total derived functor theorem 115
8. Homotopy cartesian diagrams 121

Chapter III Classical results and constructions 139
1. The fundamental groupoid, revisited 140
2. Simplicial abelian groups 145
3. The Hurewicz map 165
4. The Ex^∞ functor 182
5. The Kan suspension 189

Chapter IV Bisimplicial sets 195
1. Bisimplicial sets: first properties 196
2. Bisimplicial abelian groups 200
2.1. The translation object 201
2.2. The generalized Eilenberg-Zilber theorem 204
3. Closed model structures for bisimplicial sets 211
3.1. The Bousfield-Kan structure 212
3.2. The Reedy structure 212
3.3. The Moerdijk structure 220

4. The Bousfield-Friedlander theorem 223
5. Theorem B and group completion 233
5.1. The Serre spectral sequence 234
5.2. Theorem B 237
5.3. The group completion theorem 242

Chapter V Simplicial groups 251
1. Skeleta 252
2. Principal fibrations I: simplicial G-spaces 260
3. Principal fibrations II: classifications 265
4. Universal cocycles and $\overline{W}G$ 269
5. The loop group construction 275
6. Reduced simplicial sets, Milnor's FK-construction 285
7. Simplicial groupoids 295

Chapter VI The homotopy theory of towers 307
1. A model category structure for towers of spaces 309
2. The spectral sequence of a tower of fibrations 315
3. Postnikov towers 326
4. Local coefficients and equivariant cohomology 329
5. On k-invariants 342
6. Nilpotent spaces 350

Chapter VII Reedy model categories 353
1. Decomposition of simplicial objects 353
2. Reedy model category structures 364
3. Geometric realization 373
4. Cosimplicial spaces 376

Chapter VIII Cosimplicial spaces: applications 389
1. The homotopy spectral sequence of a cosimplicial space 390
2. Homotopy inverse limits 400
3. Completions 412
4. Obstruction theory 417

Chapter IX Simplicial functors and homotopy coherence 431
1. Simplicial functors 431
2. The Dwyer-Kan theorem 437
3. Homotopy coherence 449
3.1. Classical homotopy coherence 449
3.2. Homotopy coherence: an expanded version 451
3.3. Lax functors 454
3.4. The Grothendieck construction 455
4. Realization theorems 457

Chapter X Localization 463
1. Localization with respect to a map 464
2. The closed model category structure 478
3. Bousfield localization 489
4. A model for the stable homotopy category 492

References 503

Index 507