

CONTENTS

CHAPTER I

INTEGERS, MORE GENERAL TYPES OF NUMBER, POLYNOMIALS

	PAGE
1. Integers and Partitions	1
2. Rational and Irrational Numbers	2
3. Polynomials, Equations, Complex Numbers	3
4. Geometrical Treatment	5
5. Complex Numbers as Roots of Quadratic Equations	10
6. Algebraic Numbers and Integers	11
7. Rational Functions	12
8. Tabular and Graphical Representation	13

CHAPTER II

CONTINUITY AND EVALUATION OF POLYNOMIALS

9. Continuity	17
10. Zeros of $f(x)$, $f'(x)$, $f''(x)$	18
11. Behaviour at Infinity	19
12. The Taylor Expansion	21
13. Identities and Equations	22
14. The Practical Evaluation of a Polynomial	25
15. The Graphical Method of Lill	27
16. Horner's Method	30

CHAPTER III

THE THEORY OF RATIONAL FUNCTIONS

17. The Division of Polynomials. The G.C.M. Method	34
18. Reduction to Lowest Terms	38
19. Partial Fractions	40

	PAGE
20. Determinantal Form of Partial Fractions	47
21. The Confluent Case	48
22. The Expansion of a Rational Function	50
23. Recurring Series	53

CHAPTER IV

THE FUNDAMENTAL THEOREM OF ALGEBRA

24. Statement of the Theorem	56
25. The Product Form of an Algebraic Equation	59
26. Repeated Factors. Equal Roots	60
27. Complex Roots	63

CHAPTER V

PROPERTIES OF THE COEFFICIENTS OF AN
ALGEBRAIC EQUATION

28. The Elementary Symmetric Functions	66
29. Allied Equations and Polynomials. Zero and Infinite Roots	68
30. Further Symmetric Functions	70
31. Relations between the e and the h Functions	71
32. The Sums of Powers of the Roots	72
33. Symmetric Functions in General	75
34. Further Theory of Symmetric Functions	77

CHAPTER VI

TRANSFORMATION AND NUMERICAL SOLUTION
OF ALGEBRAIC EQUATIONS

35. Increasing or Decreasing all the Roots by the Same Amount	81
36. Removal of the Second or Third Term of an Equation	82
37. Horner's Method of Solving an Equation	83
38. Rational Roots of an Equation	88
39. Iterative Methods	90
40. Newton's Method	92

CONTENTS

xi

CHAPTER VII

THE LOCATION OF THE ROOTS OF AN EQUATION

	PAGE
41. The Significance of the Sign of a Polynomial	95
42. Newton's Rule for delimiting a Root	97
43. Relative Location of Roots of an Equation and of the Derived Equation	99
44. The Harriot-Descartes Rule of Signs	99
45. Sturm's Theorem	103

CHAPTER VIII

BINOMIAL AND RECIPROCAL EQUATIONS

46. The Binomial Equation	108
47. Euclidean Construction of the Regular Polygon	112
48. Reduction of Degree of Certain Equations. Reciprocal Equations	114

CHAPTER IX

THE CUBIC EQUATION

49. Historical and Introductory	117
50. Cardan's Solution	118
51. The Case of Equal Roots	119
52. The Ordinary Case $\Delta > 0$. The Irreducible Case $\Delta < 0$	120
53. Alternative Trigonometrical Treatment	124
54. The Equation of Squared Differences of the Roots	125
55. The General Cubic Equation	126
56. The Canonical Form of a Cubic	127

CHAPTER X

THE BIQUADRATIC OR QUARTIC EQUATION

57. Feasibility of a Solution by Radicals	130
58. Ferrari's Method of Solution	130
59. Geometrical Aspect	134
60. Canonical Forms of Even Order	135
61. Gregory's Method of Solving a Cubic or a Quartic Equation	137

CHAPTER XI

ELIMINATION OF THE UNKNOWN FROM
CONSISTENT EQUATIONS

	PAGE
62. Dialytic Elimination	140
63. Factorized Form of the Resultant	141
64. Bézout's Condensed Eliminant	142
65. Relation between the Dialytic Eliminant and Bézout's Eliminant	143
66. Case of Quantics of Unequal Degrees	145
67. Elimination and the G.C.M. Process	146

CHAPTER XII

FURTHER LIMITING AND APPROXIMATE PROCESSES

68. Explicit Formulæ for the Roots of Equations	149
69. The Irregular Case	154
70. Other Approximate Methods for Solving Equations	156
71. Newton's Limits for the Roots	157
72. Newton's Rule of Signs	157
73. Note on the Approximate Method of Newton	159