

# CONTENTS

## Preface

vii

1.	Symmetry Before Physics	1
1.1	Symmetries in Art and Architecture	1
1.2	Symmetries in Geometry	3
1.3	Symmetries in Algebra	5
2.	Groups and Their Representations	13
2.1	Examples	13
2.2	Representations	17
2.3	Quotient of Groups	19
2.4	The Fundamental Group	23
2.5	Appendix: Vector Spaces	28
2.6	Appendix: Lightning Review of Quantum Mechanics	28
3.	Lie Theory	35
3.1	Lie Algebras	35
3.2	Lie Groups	40
3.3	From Lie Groups to Lie Algebras	42
3.4	From Lie Algebras to Lie Groups*	45
4.	Rotations: $SO(3)$ and $SU(2)$	49
4.1	$SO(2)$	49
4.2	$SO(3)$	51
4.3	$SU(2)$ and its Lie Algebra $su(2)$	58

4.4	The homomorphism $R : SU(2) \rightarrow SO(3)$ . . . . .	62
4.5	$SU(2)$ as a Group Extension of $SO(3)$ . . . . .	65
5.	Angular Momentum . . . . .	69
5.1	Angular Momentum in Classical Mechanics . . . . .	69
5.2	Angular Momentum in Quantum Mechanics . . . . .	71
5.3	Representations of $so(3)$ . . . . .	73
5.4	Irreducible Representations of $SU(2)$ . . . . .	83
5.5	Spherical Harmonics . . . . .	83
5.6	The Hydrogen Atom . . . . .	84
5.7	Spin and $SU(2)$ . . . . .	87
6.	Addition of Angular Momentum . . . . .	89
6.1	Direct Products . . . . .	89
6.2	General Case of Addition of Angular Momentum . . . . .	94
6.3	The Power of Spinors* . . . . .	100
7.	Isospin and Strangeness . . . . .	103
7.1	The Atomic Nucleus . . . . .	103
7.2	Isospin . . . . .	106
7.3	The Pi Meson . . . . .	108
7.4	Hadrons . . . . .	110
7.5	Quarks . . . . .	113
7.6	The Static Quark Model . . . . .	114
7.7	K mesons . . . . .	124
7.8	$SU(3)$ . . . . .	125
7.9	Gell–Mann–Okubo Formula . . . . .	127
8.	Bosons and Fermions . . . . .	131
8.1	Partition Function . . . . .	132
8.2	The Harmonic Oscillator . . . . .	135
8.3	Free Bosons are Harmonic Oscillators . . . . .	141
8.4	Are Free Fermions some kind of Oscillators too? . . . . .	144
8.5	Beyond Free Particles: The Jaynes–Cummings Model* . . . . .	147
8.6	Heisenberg Lie Algebra . . . . .	149
8.7	Bosonic States as Polynomials . . . . .	154
8.8	The Symplectic Lie Group and its Lie Algebra . . . . .	156

8.9	The Orthogonal Lie Algebra . . . . .	160
8.10	Clifford Algebra . . . . .	162
9.	The Ising Model . . . . .	167
9.1	The Hamiltonian . . . . .	168
9.2	Transfer Matrix of the 1D Ising Model . . . . .	169
9.3	Ising Model on an $L \times 2$ Ladder . . . . .	171
9.4	The Ising Model on an $L \times M$ lattice . . . . .	172
10.	Wave Equations . . . . .	181
10.1	Lorentz Invariance . . . . .	181
10.2	Lorentz Group and Its Lie Algebra . . . . .	183
10.3	The Variational Principle for the Wave Equation . . . . .	184
10.4	The Klein-Gordon Equation . . . . .	187
10.5	Noether's Theorem . . . . .	188
10.6	Fermionic Wave Equations . . . . .	190
10.7	Variational Principle for Fermionic Wave Equations . . . . .	196
10.8	Maxwell's Equations . . . . .	197
10.9	Quantum Electrodynamics . . . . .	199
10.10	Lagrangian Formalism . . . . .	201
10.11	Yang–Mills Theory . . . . .	203
11.	Random Matrices . . . . .	209
11.1	Sources of Random Matrix Theory . . . . .	209
11.2	The Gaussian Unitary Ensemble . . . . .	214
12.	Harmonic Analysis on Finite Groups . . . . .	221
12.1	Discrete Fourier Series . . . . .	221
12.2	Non-abelian Finite Groups . . . . .	227
12.3	Central Functions . . . . .	240
12.4	An Example: The Finite Heisenberg Group . . . . .	241
13.	Harmonic Analysis on Compact Lie Groups . . . . .	251
13.1	Compact? . . . . .	251
13.2	Non-Compact Lie Groups . . . . .	252
13.3	A Tale of Two Hilbert Spaces: $l^2(\mathbb{Z})$ and $L^2(U(1))$ . . . . .	252
13.4	Invariant Integrals on Lie Groups . . . . .	263

13.5	Representations of a Compact Lie Group . . . . .	271
13.6	The Peter–Weyl Theorem . . . . .	277
14.	Quantum Groups . . . . .	283
14.1	Algebras and Co-Algebras . . . . .	284
14.2	The AntiPode . . . . .	288
14.3	Primitives, Group-Like Elements, Skew–Derivations . . . . .	291
14.4	Hopf-Modules . . . . .	292
14.5	$SL_q(2)$ . . . . .	293
15.	Euler-Arnold Dynamics . . . . .	297
15.1	The Rigid Body . . . . .	297
15.2	Euler Equations of a Fluid . . . . .	299
15.3	Euler–Arnold Equations on a Metric Lie Algebra . . . . .	301
15.4	Euler–Arnold Dynamics on $SO(3, 1)$ . . . . .	308
	<i>Bibliography</i> . . . . .	315
	<i>Index</i> . . . . .	317