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<sup>1</sup> Although calculating the rate of flow of energy in standard units is, of course, very lengthy.