

Contents

Preface	vii
Part 1. Shape derivative of minimum potential energy: abstract theory and applications	
MASATO KIMURA	1
Preface	5
Chapter 1. Shape derivative of minimum potential energy: abstract theory and applications	7
1. Introduction	7
2. Multilinear map and Fréchet derivative	10
3. Minimization problems	12
4. Banach contraction mapping theorem	15
5. Implicit function theorem	17
6. Parameter variation formulas	19
7. Lipschitz deformation of domains	23
8. Potential energy in deformed domains	28
9. Applications	30
Bibliography	37
Part 2. Geometry of hypersurfaces and moving hypersurfaces in \mathbb{R}^m for the study of moving boundary problems	
MASATO KIMURA	39
Preface	43
Chapter 1. Preliminaries	45
1. Notation	45
2. Plane curves	46
3. Parametric representation	47
4. Graph representation and principal curvatures	50
Chapter 2. Differential calculus on hypersurfaces	53
1. Differential operators on Γ	53
2. Weingarten map and principal curvatures	54
Chapter 3. Signed distance function	59

1. Signed distance function in general	59
2. Signed distance function for hypersurface	60
Chapter 4. Curvilinear coordinates	65
1. Differential and integral formulas in $\mathcal{N}^\varepsilon(\Gamma)$	65
Chapter 5. Moving hypersurfaces	69
1. Normal time derivatives	69
2. Signed distance function for moving hypersurface	72
3. Time derivatives of geometric quantities	73
Chapter 6. Variational formulas	75
1. Transport identities	75
2. Transport identities for curvatures	78
Chapter 7. Gradient structure and moving boundary problems	81
1. General gradient flow of hypersurfaces	81
2. Prescribed normal velocity motion	82
3. Mean curvature flow	82
4. Anisotropic mean curvature flow	83
5. Gaussian curvature flow	83
6. Willmore flow	84
7. Volume preserving mean curvature flow	84
8. Surface diffusion flow	85
9. Hele–Shaw moving boundary problem	86
Bibliography	89
Appendix A.	91
1. Adjugate matrix	91
2. Jacobi's formula	92
Part 3. Large time behaviour for diffusive Hamilton–Jacobi equations	
PHILIPPE LAURENÇOT	95
Chapter 1. Introduction	99
Chapter 2. Well-posedness and smoothing effects	103
1. Gradient estimates	105
2. Time derivative estimates	108
3. Hessian estimates	111
4. Existence	114
5. Uniqueness	116
Bibliographical notes	117
Chapter 3. Extinction in finite time	119
1. An integral condition for extinction	121
2. A pointwise condition for extinction	123
3. Non-extinction	124

4. A lower bound near the extinction time	126
Bibliographical notes	127
Chapter 4. Temporal decay estimates for integrable initial data: $\sigma = 1$	129
1. Decay rates	131
2. Limit values of $\ u\ _1$	134
3. Improved decay rates: $q \in (1, q_*)$	137
Bibliographical notes	139
Chapter 5. Temporal growth estimates for integrable initial data: $\sigma = -1$	141
1. Limit values of $\ u\ _1$ and $\ u\ _\infty$	142
2. Growth rates	149
Bibliographical notes	150
Chapter 6. Convergence to self-similarity	151
1. The diffusion-dominated case: $\sigma = 1$	151
2. The reaction-dominated case: $\sigma = -1$	154
Bibliographical notes	158
Bibliography	159
Appendix A. Self-similar large time behaviour	163
1. Convergence to self-similarity for the heat equation	163
2. Convergence to self-similarity for Hamilton–Jacobi equations	164
Part 4. An area-preserving crystalline curvature flow equation	
SHIGETOSHI YAZAKI	169
Preface	173
Chapter 1. An area-preserving crystalline curvature flow equation: introduction to mathematical aspects, numerical computations, and a modeling perspective	175
1. Introduction	175
2. Plane curve	175
3. Moving plane curve	178
4. Curvature flow equations	179
5. Anisotropy	181
6. The Frank diagram and the Wulff shape	183
7. Crystalline energy	187
8. Crystalline curvature flow equations	189
9. An area-preserving motion by crystalline curvature	195
10. Scenario of the proof of Theorem 1.50	196
11. Numerical scheme	198
12. Towards modeling the formation of negative ice crystals or vapor figures produced by freezing of internal melt figures	202
Bibliography	207
Index	211

Contents

Preface	5
Chapter 1. Shape derivative of minimum potential energy: abstract theory and applications	7
1. Introduction	7
2. Multilinear map and Fréchet derivative	10
3. Minimization problems	12
4. Banach contraction mapping theorem	15
5. Implicit function theorem	17
6. Parameter variation formulas	19
7. Lipschitz deformation of domains	23
8. Potential energy in deformed domains	28
9. Applications	30
Bibliography	37

Contents

Preface	43
Chapter 1. Preliminaries	45
1. Notation	45
2. Plane curves	46
3. Parametric representation	47
4. Graph representation and principal curvatures	50
Chapter 2. Differential calculus on hypersurfaces	53
1. Differential operators on Γ	53
2. Weingarten map and principal curvatures	54
Chapter 3. Signed distance function	59
1. Signed distance function in general	59
2. Signed distance function for hypersurface	60
Chapter 4. Curvilinear coordinates	65
1. Differential and integral formulas in $\mathcal{N}^\varepsilon(\Gamma)$	65
Chapter 5. Moving hypersurfaces	69
1. Normal time derivatives	69
2. Signed distance function for moving hypersurface	72
3. Time derivatives of geometric quantities	73
Chapter 6. Variational formulas	75
1. Transport identities	75
2. Transport identities for curvatures	78
Chapter 7. Gradient structure and moving boundary problems	81
1. General gradient flow of hypersurfaces	81
2. Prescribed normal velocity motion	82
3. Mean curvature flow	82
4. Anisotropic mean curvature flow	83
5. Gaussian curvature flow	83
6. Willmore flow	84
7. Volume preserving mean curvature flow	84
8. Surface diffusion flow	85
9. Hele–Shaw moving boundary problem	86
Bibliography	89

Appendix A.	91
1. Adjugate matrix	91
2. Jacobi's formula	92

93	References
94	Appendix 1. Preliminaries
95	1. Notation
96	2. Linear maps
97	3. Linear transformations and principal axes
98	4. Graphs, projections and principal axes
99	Chapter 2. Differential calculus on manifolds
100	1. Differential calculus on \mathbb{R}^n
101	2. Tangent map and principal axes
102	Chapter 3. Signed volume
103	1. Signed volume function in general
104	2. Signed volume function for hypersurfaces
105	Chapter 4. Curved surfaces
106	1. Differential and integral formulae for \mathbb{R}^n
107	Chapter 5. Moving hypersurfaces
108	1. Normal and curvature
109	2. Signed volume function for moving hypersurfaces
110	3. The evolution of geometric quantities
111	Chapter 6. Variational calculus
112	1. Transport statistics
113	2. Transport statistics for geodesics
114	Chapter 7. Geodesic structure and moving boundary problems
115	1. Geodesic gradient flow on hypersurfaces
116	2. Prescribed normal velocity motion
117	3. Mean curvature flow
118	4. Anisotropic mean curvature flow
119	5. Curvature motion flow
120	6. Willmore flow
121	7. Volume preserving mean curvature flow
122	8. Surface diffusion flow
123	9. He-Shen moving boundary problem
124	Alphabetically

Contents

Chapter 1. Introduction	99
Chapter 2. Well-posedness and smoothing effects	103
1. Gradient estimates	105
1.1. Gradient estimates: $\sigma = 1$ and $q > 1$	106
1.2. Gradient estimates: $q > 1$	107
1.3. Gradient estimates: $q = 1$	107
1.4. Gradient estimates: $q \in (0, 1)$	108
2. Time derivative estimates	108
3. Hessian estimates	111
4. Existence	114
5. Uniqueness	116
Bibliographical notes	117
Chapter 3. Extinction in finite time	119
1. An integral condition for extinction	121
2. A pointwise condition for extinction	123
3. Non-extinction	124
4. A lower bound near the extinction time	126
Bibliographical notes	127
Chapter 4. Temporal decay estimates for integrable initial data: $\sigma = 1$	129
1. Decay rates	131
1.1. Decay rates: $q = N/(N + 1)$	132
1.2. Decay rates: $q = 1$	133
2. Limit values of $\ u\ _1$	134
3. Improved decay rates: $q \in (1, q_*)$	137
Bibliographical notes	139
Chapter 5. Temporal growth estimates for integrable initial data: $\sigma = -1$	141
1. Limit values of $\ u\ _1$ and $\ u\ _\infty$	142
1.1. The case $q \geq 2$	142
1.2. The case $q \in (q_*, 2)$	144
1.3. The case $q \in [1, q_*]$	147
2. Growth rates	149
Bibliographical notes	150
Chapter 6. Convergence to self-similarity	151
1. The diffusion-dominated case: $\sigma = 1$	151

Contents

Preface	173
Chapter 1. An area-preserving crystalline curvature flow equation: introduction to mathematical aspects, numerical computations, and a modeling perspective	175
1. Introduction	175
2. Plane curve	175
3. Moving plane curve	178
4. Curvature flow equations	179
5. Anisotropy	181
6. The Frank diagram and the Wulff shape	183
7. Crystalline energy	187
8. Crystalline curvature flow equations	189
9. An area-preserving motion by crystalline curvature	195
10. Scenario of the proof of Theorem 1.50	196
11. Numerical scheme	198
12. Towards modeling the formation of negative ice crystals or vapor figures produced by freezing of internal melt figures	202
Bibliography	207
Index	211
Appendix A.	213
1. Strange examples	213
2. A non-concave curve	213
3. Anisotropic inequality—proof of Lemma 1.53—	213