

Contents

| | | |
|----------|--|-------------|
| | Foreword | <i>xi</i> |
| | Preface | <i>xiii</i> |
| | Acknowledgments | <i>xv</i> |
| | Acronyms | <i>xvii</i> |
| | About the Companion Website | <i>xix</i> |
| 1 | Introduction | 1 |
| 1.1 | OpNav Pre-history | 2 |
| 1.1.1 | Maritime Celestial Navigation | 2 |
| 1.1.2 | Spacecraft OpNav before Sputnik | 16 |
| 1.2 | The Rise of Radio Navigation | 18 |
| 1.3 | OpNav in Crewed Spaceflight | 19 |
| 1.4 | OpNav in Robotic Spaceflight | 20 |
| 1.4.1 | Missions to the Outer Planets | 22 |
| 1.4.2 | Missions to Asteroids and Comets | 25 |
| 1.5 | Terrain Relative Navigation | 28 |
| 2 | Mathematical Foundations | 31 |
| 2.1 | Set Theory and Algebraic Structures | 31 |
| 2.1.1 | Sets | 31 |
| 2.1.2 | Number Systems | 32 |
| 2.1.3 | Numeral Systems | 35 |
| 2.1.4 | Set Operations | 37 |
| 2.1.5 | Groups, Rings, and Fields | 40 |
| 2.2 | Vector Spaces and Linear Algebra | 43 |
| 2.2.1 | Vector Spaces | 43 |
| 2.2.2 | Inner Products and Norms | 44 |
| 2.2.3 | Orthogonality and Linear Independence | 45 |
| 2.2.4 | Basis Vectors | 46 |
| 2.2.5 | Reference Systems and Reference Frames | 47 |
| 2.2.6 | Matrices | 48 |
| 2.2.7 | Linear Transformations and Group Actions | 49 |
| 2.3 | Change of Basis and Orthogonal Matrices | 51 |

| | | |
|----------|--|-----------|
| 2.3.1 | Representing a Vector with Different Bases | 51 |
| 2.3.2 | Attitude Transformations | 54 |
| 2.3.3 | Orthogonal Procrustes Problem | 56 |
| 2.3.4 | Weighted Orthogonal Procrustes Problem | 60 |
| 2.4 | Attitude Parameterizations | 60 |
| 2.4.1 | Euler's Rotation Theorem | 60 |
| 2.4.2 | Euler Angle and Euler Axis | 61 |
| 2.4.3 | Rotation Vectors | 63 |
| 2.4.4 | Attitude Quaternions | 64 |
| 2.4.5 | Rodrigues Parameters and Modified Rodrigues Parameters | 68 |
| 2.5 | Geometric Algebra | 70 |
| 2.5.1 | Grassmann Algebra | 71 |
| 2.5.2 | Clifford Algebra | 75 |
| 2.6 | Polynomials | 78 |
| 2.6.1 | Quadratics and Cubics | 78 |
| 2.6.2 | Polynomials in a Single Variable | 79 |
| 2.6.3 | Root Finding for Polynomials in Single Variable | 82 |
| 2.6.4 | Polynomials in a Single Variable as a Vector Space | 85 |
| 2.6.5 | Polynomials in Many Variables | 86 |
| 2.6.6 | Algebraic Varieties | 88 |
| 2.7 | Conics (A First Encounter) | 88 |
| 2.7.1 | Circles | 90 |
| 2.7.2 | Ellipses | 92 |
| 2.7.3 | Parabolas and Hyperbola | 96* |
| 3 | Projective Geometry | 97 |
| 3.1 | Perspective and the Pinhole Camera Model | 98 |
| 3.1.1 | Camera Obscura | 98 |
| 3.1.2 | The Image Plane | 101 |
| 3.1.3 | Pinhole Camera Model | 103 |
| 3.2 | Rules of Perspective Projection | 104 |
| 3.3 | An Axiomatic Perspective | 110 |
| 3.3.1 | The Axioms of Euclidean Geometry | 111 |
| 3.3.2 | The Axioms of Projective Geometry | 111 |
| 3.3.3 | Duality | 112 |
| 3.3.4 | Perspectivities and Projectivities | 113 |
| 3.3.5 | Desargues's Theorem | 115 |
| 3.3.6 | Harmonic Sets | 116 |
| 3.3.7 | Fundamental Theorem of Projective Geometry | 117 |
| 3.4 | An Algebraic Perspective | 120 |
| 3.4.1 | Projective Space, \mathbb{P}^n | 120 |
| 3.4.2 | Homogeneous Coordinates | 121 |
| 3.4.3 | Points, Lines, and Planes | 123 |
| 3.4.4 | Conics | 138 |
| 3.4.5 | Quadrics | 150 |
| 3.5 | Invariants | 155 |
| 3.5.1 | Cross-ratio in \mathbb{P}^1 | 156 |

| | | |
|----------|---|------------|
| 3.5.2 | Cross-ratio in \mathbb{P}^2 | 159 |
| 3.5.3 | View Invariants | 162 |
| 3.6 | Two-dimensional Transformations | 163 |
| 3.6.1 | Euclidean Group | 166 |
| 3.6.2 | Similarity Group | 168 |
| 3.6.3 | Affine Group | 170 |
| 3.6.4 | Projective Linear Group | 171 |
| 4 | Time, Reference Frames, and Orbits | 181 |
| 4.1 | Time and Angle | 181 |
| 4.1.1 | Subdivision of the Day and Circle | 181 |
| 4.1.2 | Angle Measurements | 182 |
| 4.1.3 | SI Second, International Atomic Time, and Terrestrial Time | 186 |
| 4.1.4 | Julian Dates | 187 |
| 4.2 | Equinoxes and Solstices | 188 |
| 4.3 | Celestial Reference Frames | 192 |
| 4.3.1 | Coordinate Systems on the Celestial Sphere | 192 |
| 4.3.2 | Dynamically-defined Reference Systems | 195 |
| 4.3.3 | Kinematically-defined Reference Systems | 197 |
| 4.4 | Days, Calendars, and Civil Time | 198 |
| 4.4.1 | Day, Month, and Year | 198 |
| 4.4.2 | Universal Time | 203 |
| 4.4.3 | Civil Time | 205 |
| 4.5 | Two-body Orbital Mechanics | 205 |
| 4.5.1 | Equations of Motion | 209 |
| 4.5.2 | Conservation of Energy | 211 |
| 4.5.3 | Conservation of Angular Momentum | 212 |
| 4.5.4 | The Two-body Trajectory Equation | 214 |
| 4.5.5 | Orbital Period | 217 |
| 4.5.6 | Perifocal Frame and Orbital Elements | 217 |
| 4.5.7 | Orbital Hodograph | 219 |
| 4.5.8 | An Algebraic View of Orbits | 221 |
| 4.6 | Dissemination of Celestial Geometry | 227 |
| 4.6.1 | Historical Astronomical Tables and Ephemerides | 228 |
| 4.6.2 | Modern Astronomical Tables and Ephemerides | 230 |
| 5 | Astrometry and Star Catalogs | 233 |
| 5.1 | The Propagation of Light | 234 |
| 5.1.1 | Maxwell's Equations | 234 |
| 5.1.2 | The Wave-like Behavior of Light | 235 |
| 5.1.3 | Wavefronts and Rays | 239 |
| 5.1.4 | Refraction, Snell's Law, and Fermat's Principle | 239 |
| 5.2 | Asterisms and Constellations | 244 |
| 5.3 | Classical Star Catalogs | 245 |
| 5.3.1 | Babylonian Star Catalogs | 245 |
| 5.3.2 | Classical Greek Star Catalogs | 246 |
| 5.3.3 | Visualizations of the Celestial Sphere in Classical Antiquity | 249 |

| | | |
|----------|---|------------|
| 5.3.4 | Other Catalogs in Antiquity | 253 |
| 5.3.5 | European Star Catalogs and Celestial Cartography | 255 |
| 5.3.6 | Star Catalogs in the 20th Century | 258 |
| 5.4 | Modern Astrometry and Star Catalogs | 259 |
| 5.4.1 | Modern Astrometric Models | 259 |
| 5.4.2 | USNO Catalogs | 268 |
| 5.4.3 | Catalogs from Space-based Scanning Astrometry | 270 |
| 5.5 | Stochastic Catalogs | 276 |
| 5.5.1 | Uniform Sampling on the Sphere | 277 |
| 5.5.2 | Distribution of Inter-star Angles on the Sphere | 278 |
| 5.6 | Theory of Relativity | 279 |
| 5.6.1 | Principle of Relativity | 280 |
| 5.6.2 | Special Theory of Relativity | 281 |
| 5.6.3 | Stellar Aberration | 287 |
| 5.6.4 | General Theory of Relativity | 291 |
| 5.6.5 | Practical Computation of Gravitational Light Deflection | 300 |
| 6 | Radiometry and Photometry | 303 |
| 6.1 | Electromagnetic Spectrum | 304 |
| 6.2 | Photons and Quantum Electrodynamics | 305 |
| 6.3 | Radiometric Units of Measure | 308 |
| 6.3.1 | Power, Flux Density, and Irradiance | 308 |
| 6.3.2 | Solid Angles | 312 |
| 6.3.3 | Intensity and Radiance | 316 |
| 6.4 | Blackbody Radiation | 318 |
| 6.5 | Apparent Magnitude | 322 |
| 6.6 | Photometric Systems | 324 |
| 6.7 | Transmittance and Optical Depth | 325 |
| 6.8 | Single-scattering Phase Function | 328 |
| 6.8.1 | Rayleigh Phase Function | 329 |
| 6.8.2 | Henyey–Greenstein Phase Function | 330 |
| 6.9 | Reflectance Models | 332 |
| 6.9.1 | Reflectance and Photometric Functions | 332 |
| 6.9.2 | Conservation of Energy | 333 |
| 6.9.3 | Helmholtz Reciprocity | 334 |
| 6.9.4 | Fresnel Equations (Specular Reflection) | 335 |
| 6.9.5 | Lambertian Model (Diffuse Reflection) | 341 |
| 6.9.6 | Lommel–Seeliger Model | 343 |
| 6.9.7 | Chandrasekhar Model | 347 |
| 6.9.8 | Hapke Model | 351 |
| 6.9.9 | Lunar-Lambert Model | 357 |
| 6.10 | Reflectance Models for Rough Planetary Surfaces | 358 |
| 6.10.1 | Modeling Rough Surfaces | 359 |
| 6.10.2 | Geometric Attenuation Factor | 360 |
| 6.10.3 | BRDFs for Rough Surfaces | 373 |
| 6.10.4 | Torrance–Sparrow Model | 374 |
| 6.10.5 | Oren–Nayar Model | 376 |

| | | |
|----------|--|------------|
| 6.11 | Reflectance Model Comparisons | 377 |
| 6.12 | Resolved Photometry | 378 |
| 6.12.1 | Terrain Models | 379 |
| 6.12.2 | Ray Tracing | 384 |
| 6.13 | Unresolved (Disk-integrated) Photometry | 391 |
| 6.13.1 | Spherical Celestial Bodies | 392 |
| 6.13.2 | Disk-integrated Albedos | 398 |
| 6.13.3 | Apparent Magnitude of a Celestial Body | 402 |
| 6.13.4 | Lightcurve Analysis | 406 |
| 7 | Camera Hardware and Models | 407 |
| 7.1 | Overview of Camera Systems | 407 |
| 7.1.1 | Camera Obscura (Redux) | 407 |
| 7.1.2 | Framing Cameras | 411 |
| 7.1.3 | Scanning Cameras | 413 |
| 7.2 | Light Baffles | 413 |
| 7.3 | Optical Assembly | 419 |
| 7.3.1 | Mirrors (Reflecting Elements) | 419 |
| 7.3.2 | Lenses (Two-surface Refracting Elements) | 425 |
| 7.3.3 | Systems of Mirrors and Lenses | 437 |
| 7.3.4 | Stops and Pupils | 439 |
| 7.3.5 | Seidel Aberrations | 441 |
| 7.3.6 | Point Spread Function | 444 |
| 7.3.7 | Optical Distortion Model | 448 |
| 7.4 | Image Sensors | 451 |
| 7.4.1 | Digital Images | 451 |
| 7.4.2 | Photodetectors | 462 |
| 7.4.3 | Focal Plane Arrays | 470 |
| 7.4.4 | Dynamic Vision Sensors (Event Cameras) | 483 |
| 7.5 | Camera and Optical Instrument Design | 485 |
| 8 | Navigating with Stars | 489 |
| 8.1 | Modeling Stars in Digital Images | 490 |
| 8.2 | Star Detection and Centroiding | 493 |
| 8.2.1 | Star Detection | 493 |
| 8.2.2 | Idealized Star Centroiding | 496 |
| 8.2.3 | Practical Centroiding Methods | 500 |
| 8.3 | Attitude Determination | 501 |
| 8.3.1 | Calibrated vs. Uncalibrated Attitude Determination | 501 |
| 8.3.2 | Calibrated Attitude Determination: Wahba's Problem | 502 |
| 8.3.3 | Uncalibrated Attitude Determination | 507 |
| 8.3.4 | Attitude Covariance | 516 |
| 8.4 | Star Identification | 519 |
| 8.4.1 | A Framework for Automated Star Identification | 520 |
| 8.4.2 | Asterism Invariants | 520 |
| 8.4.3 | Asterism Catalog Curation | 534 |
| 8.4.4 | Database Management and Query | 537 |

| | | |
|-----------|--|------------|
| 8.4.5 | Systematic Construction of Image Asterisms | 543 |
| 8.4.6 | Asterism Validation | 544 |
| 8.5 | Velocity Estimation from Stellar Aberration | 544 |
| 8.5.1 | Measurement Model | 545 |
| 8.5.2 | Velocity-based Orbit Determination | 548 |
| 9 | Celestial Navigation | 551 |
| 9.1 | Global Shape of Self-gravitating Bodies | 551 |
| 9.1.1 | Nonrotating Bodies | 552 |
| 9.1.2 | Slowly Rotating Bodies | 553 |
| 9.1.3 | Arbitrarily Rotating Bodies | 556 |
| 9.1.4 | Spherical Harmonics | 565 |
| 9.2 | Images of Ellipsoidal Celestial Bodies | 569 |
| 9.2.1 | Analytic Horizon Projection | 570 |
| 9.2.2 | Analytic Terminator Projection | 574 |
| 9.2.3 | Photocenter Offset for Unresolved Objects | 577 |
| 9.3 | Horizon-based Position Estimation | 580 |
| 9.3.1 | Limb Scanning | 581 |
| 9.3.2 | Limb Localization | 583 |
| 9.3.3 | Non-iterative Horizon-based OpNav | 590 |
| 9.4 | Horizon-based Attitude Determination | 601 |
| 9.5 | Triangulation | 603 |
| 9.5.1 | Trigonometric Solutions | 606 |
| 9.5.2 | Triangulation with Noisy Measurements | 610 |
| 9.5.3 | Celestial Triangulation | 613 |
| 9.6 | Navigation Filters | 619 |
| 9.6.1 | State Selection and Dynamical Model | 620 |
| 9.6.2 | State and Covariance Propagation | 623 |
| 9.6.3 | State and Covariance Update | 628 |
| 9.6.4 | Measurement Models | 631 |
| 10 | Terrain Relative Navigation | 637 |
| 10.1 | Landmarks | 637 |
| 10.1.1 | Craters | 638 |
| 10.1.2 | Features | 646 |
| 10.2 | Map-Free TRN | 648 |
| 10.2.1 | Visual Odometry Measurements | 649 |
| 10.2.2 | Filter Processing of Visual Odometry Measurements | 653 |
| 10.3 | Map-based TRN | 655 |
| 10.3.1 | Monocular Position Estimation | 655 |
| 10.3.2 | Monocular Pose Estimation: Perspective-n-point Problem | 657 |
| 10.3.3 | Filter Processing of Landmark Observations | 660 |
| | References | 667 |
| | Index | 709 |